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"GREEN" INNOVATION WITHOUT A HANDLE: FINDING EFFECTIVE INNOVATION POLICIES

Master Thesis

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Contents

Abstract

The Earth's climate is warming at a rapid rate. To address this issue, emissions need to be reduced. In an economy of investing and producing firms, a policymaker might be unable to distinguish investments in cost reductions from "green" investments in emissions reductions. To investigate this issue, I propose and subsequently solve a three-stage Cournot model. In this model a policymaker first sets a tax or subsidy on investments that applies to both investments equally and an emissions tax. In the second stage firms invest into both reductions and in the third stage they produce their goods. I use the model to answer the question, whether a policymaker with no direct handle on "green" investments can steer the economy away from the externality of climate change. Beside the externality of climate change, the model economy exhibits the inefficiencies of market power and over-investment that cannot be fully addressed by the policymaker. However, even though the policymaker has no direct influence on the individual "green" investments, they can still control their amount through the combination of investment tax or subsidy and the emissions tax.

1. Introduction

The Earth's Climate is warming at an unprecedented rate, posing a threat to planetary health and human well-being (IPCC, 2023). The problem is widely known for more than 30 years and commitments have been made to address the issue (UN General Assembly (43rd Sess.: 1988-1989), 1989; UNFCCC, 2015). While these commitments are made on the global stage, climate action is enforced on the local stage (Putnam, 1988). The most recent example of these two stages are the nationally determined contributions (NDCs)[1] set by countries in accordance with the Paris agreement, which need to be enforced on the national level. However, goals set by these NDCs often fall behind the targets of the Paris agreement and most countries' domestic actions are not even consistent with their stated goals (Boehm et al., 2022). Assuming that this inconsistency is unintentional, the question is how to improve upon it.

Much domestic action is enforced by policies. These policies are implemented by policymakers who face a diverse economy that can be influenced in different ways. Because some ways may be more effective than others, policymakers need a good

[1] A mechanism introduced in the Paris agreement that allows countries to define the climate targets on their own. These targets need to increase over time (ratchet up) and are legally binding.

understanding of policy instruments and mechanisms relevant to their society. When thinking about the climate problem from the perspective of a policymaker, greenhouse gas emissions become an externality owing to the emitters who do not fully take into account the damages they cause (Nordhaus, 1991). Hence, an unregulated economy is not able to solve the climate problem.

A policymaker has the power to who steer firms in their transition towards a clean future using adequate plans, policy tools and clear communication (Farmer et al., 2019; Stern & Valero, 2021). Among different climate policies, the most prominent is putting a price on carbon through a carbon tax or an emissions trading scheme (Convery, 2009; Elkins & Baker, 2001). Other policies set renewable energy standards or restrict the supply of fossil fuels (Green & Denniss, 2018; Rausch & Mowers, 2014).

When a firm is faced with a climate policy, making further emissions of climate gases unattractive, it has to restructure its production processes to decrease their emissions intensity. Multiple processes have been proposed to decrease the carbon intensity of a producing firm, ranging from efficiency gains over novel processes towards capturing the emissions or even filter them out of the air (Greening et al., 2000; Kätelhön et al., 2019; Olajire, 2010). While these processes differ vastly in their effectiveness, applicability and permanence, they share one thing in common: they are novel to the firm and it needs to invest money to make them work. Another part of running a firm is the a constant pressure to reinvest and increase profitability to stay competitive. Such measures might include investments into research and development or optimising processes through applying them often enough (learning by doing) (Lewis & Nemet, 2021; Szücs, 2018).

A policy maker might hand out subsidies to induce firms on conducting more research. This might be in the form of state-owned universities that are open to cooperation with firms or tax cuts for money invested into R & D efforts (Becker, 2015; Szücs, 2018). However, it might be hard to control the exact purpose of the money. Processes optimising production might go hand in hand with decreases in production costs or lead to more emissions due to rebound effects (Greening et al., 2000). Therefore, the distinction between investments into carbon reductions and investments into production costs reduction can be infeasible for a policymaker. The question then becomes how to steer the economy towards more investments into emissions reductions when the policy tool can only ever address investments as a whole.

Therefore these research questions are to be answered here

1. What variables influence an individual firms decision to invest into costs or emissions reductions?

2. Will and if so when will a benevolent policy maker have different preferences than the individual firm?

3. What are the policies to guide the individual firm towards the socially optimal distribution and is it able to achieve that distribution?

To answer these questions, a two-stage Cournot model is formulated that introduces a production economy with a first stage of investment decisions and a second stage deciding on the output. For the sake of simplicity the investments contain no spillovers and all firms are homogeneous. The model shows a clear analytical solution highlighting the influences of the taxes and subsidies imposed upon the firms.

In a second calculation, a social planner enters the stage to calculate the optimal allocations of goods with no games played. The optimum is then enforced in the market by defining a social welfare function, inserting the market solutions and optimising for taxes. While the equation being a third order polynomial is analytically solvable, its solution would span several pages and defies interpretation. It is therefore interpreted in the light of multiple numerical illustrations.

Calculations yield a clearly defined carbon tax with linear dependence on the investment policy if defined. The investment policy only shows an analytical solution in the perfect competition case where it disincentivises investments on behalf of more production. In combination with an investment policy the carbon tax begins at half of the damages caused by emissions and quickly starts to approach the total damages for more firms which it reaches for perfect competition. In the absence of an investment policy, this shape is surprisingly reversed, showing a tax equal to the full carbon damages for a monopoly and then asymptotically approaching half of the damages with increasing competition.

2. Literature Review

In the economics literature, there is broad coverage of concerns of climate change, ranging from discussions of generational equity (Nordhaus, 2007; Weitzman, 2007), to the analysis of possible transition paths to a net zero society (Farmer et al., 2019; Stern & Valero, 2021) and the effects of market power on renewable utility (Dorer, 2022; Reichenbach & Requate, 2012). The concern of this work is on the effect of market power in an economy of innovative producing firms guided by a benevolent policy maker.

Therefore, this literature review investigates the strands of literature on innovative processes, oligopolistic competition and addressing occurring inefficiencies through adequate policies.

2.1. Modelling Innovative Processes

Faced with the question, how to represent innovation in an analytic model, the economic literature often considers it as a form of learning. While there are other interesting types of innovation such as disruptive innovations, they are much harder to predict or model and therefore not considered in this work (Wilson, 2018). Mentions of learning date back to the 1930s with the specific case of airplane construction costs and the early 1960s observe an adaptation by Arrow for the economics context (Arrow, 1962; Wright, 1936). In the late 1990s, this learning was also applied in low-carbon energy analyses and has seen wide adoption in many low-carbon economy settings since (Lewis & Nemet, 2021; Wene, 2000). While this is limiting innovation to a special case, the ways to represent, study and estimate learning are manifold.

Learning can be studied through learning curves, expert elicitations, patent analysis, engineering decomposition and policy intervention studies (Lewis & Nemet, 2021). Since it is often most intuitive to use learning curves for analytical models, this is the representation of learning chosen in this work. The topic of climate economics has seen the application of learning curves to a variety of fields, including energy economics (Zhang et al., 2020), "*green*" innovation (Lambertini et al., 2017) and the evolution of carbon capture and storage (Riahi et al., 2004).

These learning curves link the decrease in production costs or the increase in efficiency to experience in production, research and development (R & D) expenditures or other economic factors by a functional relationship. Exact functional forms of these curves vary and their actual analytical dependency can be for instance on expenditures towards cost decrease, time spent producing or production capacity. These factors can be present individually in so-called single factor learning curves or multiple factors yield influence at once. Since multi-factor learning curves are often used to untangle learning by doing and learning by research (Rubin et al., 2015), their use is not beneficial to the efforts of this study. Here, I only address a single learning investment which can be time, resources, knowledge, money or a combination of all, depending on context.

A large strand of research on learning is concerned with the fact that knowledge is not an easily contained asset but spreads (Dorer, 2022; Fischer & Newell, 2008). If one firm finds a specific innovation, there will be competitors copying that innovation without the need for similar expenditures. These so-called spillovers - if accounted for - change the incentives to invest for the individual firm (usually away from innovation). They can also be represented in learning models but add another layer of complexity and are therefore only mentioned here for the sake of completion (Dorer, 2022).

Considering an innovation policy, a policymaker can often not control the dedication of funds for publicly funded research projects such as universities. Therefore the policy designed to steer investments into innovation does not distinguish between innovation aimed at emissions reduction and innovation aimed at costs reduction. To my knowledge this research while being a type of asymmetric information has not been carried out before. The question becomes whether the policy maker can steer the dedication of funds in this setting towards the social optimum.

2.2. Effects of Competition

In an economic environment which is subject to changes in market power (De Loecker et al., 2020), the study of learning under different conditions of market power is also of interest. Therefore, the model also focuses on competition among firms. When firms face little pressure from competition, their behaviour differs from firms who are faced with strong competition. In the event of lacking competition, firms take into account the effect their production quantity has on the market price. Thereby a firm increases its profits but lowers overall welfare by under-supplying.

To depict market power, I use a model of Cournot competition introduced by Friedman (1983). This models a non-cooperative competition of firms taking into account the decisions of their competitors. Then, each firm takes into account the effect of its output decisions on the equilibrium price when maximising profits. Modelling this behaviour, Cournot competition can represent monopolies, duopolies, oligopolies and competitive markets.

The effect of competition on innovative processes is not easily understood. The middle of the last century saw a disagreement between leading economists. On the one hand, Schumpeter (1942) argued that more concentrated firms lead to a safer investment environment necessary to finance innovations. On the other hand, Arrow (1962) argued that big companies are often too stiff in their processes to keep up with the fast-paced changes to adapt to new innovations.

These comments refer to factors in firms that are hard to measure. Maybe it is for this fact, that the question whether competition hinders or helps innovative processes is still recent. One paper to investigate this topic is written by Lambertini et al. (2017). They find an "*inverted U-relationship*" between the aggregate amount of research and the number of firms competing in an economy. In their Cournot model of competing firms with an option to invest in R & D under varying terms of spillovers, the authors find that from a maximum for a given number of firms, investments decrease with increasing and decreasing competition.

Lambertini et al. (2017) found that this relationship between investments and competition breaks down in case of no spillovers in favour of a steadily increasing relationship. Another paper by Ulph and Ulph (2007) employs a model of countries with Cournot competition and environmental and technology policies. Looking at an increasing number of countries and thus firms, competing in the market without R & D spillovers, they find that firms commit to strategic over-investment. This means firms invest too much into R & D in contrast to the actual gains they derive from that investment. While not necessarily indicating decreasing emissions with increasing competition they state that there is a point from which on firms invest too much in case of no research. An interesting question I try to answer with this research, is whether the inverted-U relationship quoted by Lambertini et al. (2017) can also be observed without spillovers. Lambertini et al. states that this model will not show an inverted-U relationship without spillovers. However, a benevolent policy maker addressing the issue of over-investment quoted by Ulph and Ulph (2007) might reintroduce it.

2.3. Policies for Inefficiencies

As already outlined in the previous subsection, the non-perfect competition introduced through the Cournot model poses an inefficiency to the general welfare. Another inefficiency, I expect my model to exhibit is the one of over-investment as presented by the aforementioned Ulph and Ulph (2007). As suggested by the authors, I could exchange this inefficiency for the more realistic inefficiency of spillovers but leave them out for simplicity. The last inefficiency is the externality of climate change. A firm polluting the environment by releasing greenhouse gases is not taking into account their damages to the whole society and thus causes an externality.

If a benevolent policymaker addresses these inefficiencies they must put in place policies. There are two policies I consider here, which will be introduced in more detail in the next section: a policy on investments and a policy on carbon emissions. Most studies - empirical and analytical - suggest that investments into R & D should be publicly incentivised through subsidies (Becker, 2015; Davidson & Segerstrom, 1998; Fischer & Newell, 2008) rather than disincentivised through taxation (Ulph & Ulph, 2007). Examples of investment policies include feed-in-tariffs or publicly funded universities

(Reichenbach & Requate, 2012; Szücs, 2018). Proposed policies to reduce greenhouse gas emissions, on the other hand, vary in their function and to whom they apply, ranging from taxation or emissions trading schemes to supply-side policies and emissions standards (Elkins & Baker, 2001; Green & Denniss, 2018; Rausch & Mowers, 2014). However, I focus on taxation and research shows that the policy should be a tax rather than a subsidy (in analytical settings as well as the actual implementations) (Convery, 2009; Fischer & Newell, 2008).

Studies on these policies have been conducted individually or several in combination where the order of policies is not necessarily clear (Reichenbach & Requate, 2012). The study of Fischer and Newell (2008) finds that pricing the greenhouse gas emissions is the most important policy which can be supplemented by an R & D policy. Their study employs a model of renewable and fossil-fuel energy providers with various policies that innovate with spillovers. This contrasts the work of Dorer (2022) who finds that the investment subsidy is the single most important policy measure in reducing emissions. In another work by Reichenbach and Requate (2012), the authors find that in a Cournot model of the energy market with fossil-fuelled utilities, renewables and renewable equipment providers, the ideal R & D subsidy performs worse than the first best optimum. In their model the carbon tax is set exogenously too low, thus highlighting the importance of that tax. Especially in the context of a carbon price, the effects of the time horizon or uncertainty on said price have also been discussed (Sterner & Persson, 2008; Weitzman, 2007).

A review by Lambertini (2017) summarises the different types of "green" innovation considered in interaction with market effects. In addition to stimulating innovation through policy instruments such as emission taxes, stimulation through consumer preference is also considered. The analysis concludes that consumer preferences can at least partially substitute for the effect of policy instruments, but cannot make a clear statement about the extent of the instruments nor to what degree they can be substituted by consumer preferences. His survey of the existing literature on the topic brought no clear hierarchy of policies.

3. Model Definition

Here, I define the model for my analytical examinations as a three stage non-cooperative game of producing innovative firms under Cournot competition with a benevolent policy maker. The purpose of the model is to identify the socially optimal taxes / subsidies for a policymaker to address the market inefficiencies of climate damages, strategic over-investment and market power. The model is introduced in three parts: First, the decentralised market setting is introduced; second, the policy instruments to influence the allocation decisions of the firms are explained; and third, a social welfare function to estimate overall social welfare is stated.

3.1. Decentralised Market Setting with n Identical Firms

The model consists of a set of n producing firms $\mathcal{F}$ that sell their goods q_i on a market at price $p\,(Q)$ where $Q = \sum_{i \in \mathcal{F}} q_i$. The price is determined by the total amount of output through the linear inverse demand function:

$$p(Q) = p_0 - \eta\, Q\,. \tag{1}$$

Every firm faces production costs $\gamma_i(q_i) = c_i q_i$ and emissions $\epsilon_i(q_i) = e_i q_i$ where c_i and e_i are the unit costs and unit emissions respectively. A firm has the option to invest to reduce its unit costs or unit emissions before beginning production. The relation between money invested and unit costs or unit emissions reduced is given by the functions

$$c_i(x_i^c) = c_i^0 \exp\left(-\ell_i^c x_i^c\right)\,, \tag{2}$$

$$e_i(x_i^e) = e_i^0 \exp\left(-\ell_i^e x_i^e\right)\,. \tag{3}$$

The superscript zero refers to initial values and the ℓ's denote the improvement rates.

While other publications represent investments differently (Lambertini, 2017), the negative exponential functional form used here was decided on behalf of its monotonically decreasing marginal benefits. This reflects the often reported increasing price of carbon over time which implies that the relatively easy to abate emissions will be addressed before the ones that are harder to abate[2] (Kesicki & Strachan, 2011).

The part of the game concerned with the market is structured as follows: First, all firms simultaneously take decisions on the investments they want to make and second, again all simultaneously decide on their individually optimal amount of production. The setup hence turns into a two-stage game of firms optimising their profits. Compared to a

[2]Although the idea presented here seems quite convincing, it does not fully address the possibility of innovation and the interdependence between emission reduction efforts and should therefore be treated with caution.

central policy maker that decides on all the firms' decisions simultaneously this setting, is further referred to as the *decentralised market*.

3.2. Policy Instruments

As mentioned in the literature review in Section 2, there are some inefficiencies present in the model. If a policymaker wants to steer this market away from these inefficiencies, they need to employ some policy instruments to address these inefficiencies. The instruments modelled here, are an emissions tax, τ_e; and an investment tax / subsidy, τ_c. The general assumption made here is that all firms in the economy accept policies as givens and do not take into account the effect of their allocations on the policy instrument. Further, I call a policy instrument a tax if its value is positive and a subsidy if its value is negative. To balance the policy maker's budget, the sum of taxes and subsidies are either reimbursed to the firms if the taxes outweigh subsidies or paid as an additional tax by the firms if subsidies outweigh taxes. This balancing is done with a lump-sum, denoted by the variable w.[3]

3.2.1. Emissions Tax

As explained before, emissions are an economic inefficiency due to the externalities that the polluting firm exerts on society. Thus, the carbon tax τ_e introduces a cost on carbon, a firm previously did not need to pay. Among the most prominent real-world examples of this highly stylised tax is the European Union's emissions trading scheme (EU-ETS) (Convery, 2009). Although the pricing happens by auctions and secondary market transactions, the main idea remains the same. Under perfect conditions, a tax and a permissions trading scheme should produce equal outcomes in allocation (Elkins & Baker, 2001). For this highly stylised model, the exact type of implementation of a price on emissions is not relevant.

3.2.2. Investment Tax / Subsidy

Since investments in research might deviate from the socially optimal level, a policymaker might want to guide them. This is achieved by the introduction of an investment tax / subsidy τ_c. Among policy instruments observed are research subsidies but also funding for universities through university-industry cooperations (Szücs, 2018). The model does

[3] There are different models for the reimbursement or recycling in the form of energy-efficiency subsidies (Bourgeois et al., 2021; Boyce, 2018; Mildenberger et al., 2022) but since all firms are homogeneous in the model every distribution scheme would lead to the same final allocation. Therefore, the actual distribution is not of importance here.

not fully capture these instruments, since for example, the university-industry cooperation would also produce non-proprietary knowledge not modelled here.

In my model the policymaker cannot control the dedication of the investment tax / subsidy. While they can attribute its quantity, they have no direct control over whether funds are dedicated to unit cost decrease or emissions reductions. This is easily illustrated in the aforementioned example of university-industry cooperation where the subsidy would be the state funding public universities. Whether the research done by these universities helps a firm in reducing unit costs or emissions is not controlled by the policymaker. By distributing money to universities, they can decide which topics to address but whether the research generates costs or emissions reductions is unclear. While prior research investigated the impacts of consumer behaviour or product differentiation (Lambertini, 2017), the setting in which the policymaker has no control over the dedication of funds has to my knowledge not been explored. It thus poses a key assumption of this model, that the policymaker cannot distinguish between unit cost improvements and emissions intensity reductions in his investment policy tool.

Having established the setting in which products are traded, the costs and ways to reduce them as well as taxes / subsidies a firm might face, the firms' profits can be described by the function

$$\pi_i = (p(Q) - c_i - \tau_e e_i)q_i - (x_i^c + x_i^e)(1 + \tau_c) + w\,, \qquad \forall i \in \mathcal{F}\,. \tag{4}$$

3.3. Social Welfare Function

While the firms and their profits pose an important part of the economy, the society is more than the producing firms. Finding a function to represent the social welfare of a society has been the concern of many economists. Most noticeably Arrow (1950) in his *Possibility Theorem* stated that five reasonable conditions upon the welfare function leads to a contradiction, thus showing that aggregating the individual preferences of a society into a single function always hurts one of the proposed conditions. Since gross domestic products are still the most widespread metrics to measure welfare[4] and equality is not a concern for this simple model, the use of money as an approximation for a measure of well-being seems justified.

The policymaker thus needs to account for three contributions. The first is the consumer surplus (CS), namely the difference between what consumers were willing to pay for a product compared to what they actually paid.[5] The second is the producer surplus (PS), which is the sum of all individual firms' profits. The third are the damages (D),

[4]Alternatives considering government services, unpaid work and distributional equality have been proposed (Aitken, 2019).

[5]While Ulph and Ulph (2007) propose that the inclusion of consumer surplus will not affect their results, I am not convinced whether their argument applies here and therefore consider it.

which are the total emissions of the economy multiplied by the damages α they cause. In this setting, the assumption of a linear functional form for climate damages is a good approximation, especially since the model involves no temporal component. Put into equations these three components are

$$CS = \int_0^Q [p_0 - p(Q)] \mathrm{d}Q = \frac{1}{2} [p_0 - p(Q)] \, Q = \sum_{i \in \mathcal{F}} \frac{1}{2} [p_0 - p(Q)] \, q_i \,, \tag{5}$$

$$PS = \sum_{i \in \mathcal{F}} \pi_i = \sum_{i \in \mathcal{F}} [(p(Q) - c_i) q_i - x_i^c - x_i^e] \,, \tag{6}$$

$$D = \sum_{i \in \mathcal{F}} \alpha e_i q_i \,. \tag{7}$$

which can then be combined into a single function

$$U = CS + PS - D = \sum_{i \in \mathcal{F}} \left[\left(\frac{1}{2} (p_0 + p(Q)) - c_i - \alpha e_i \right) q_i - x_i^c - x_i^e \right] \,. \tag{8}$$

To find socially optimal values for the policies the policymaker inserts the results from the decentralised market competition into the social welfare function and maximises with respect to the policies. Since this maximisation occurs before firms decide on investments and output, this thus represents the first stage of the game which is now complete. The next chapter shows the solution to the three stages.

4. Solving the Model

The model proposed in Section 3 defines a three-stage game. The model is solved by calculating a subgame-perfect Nash equilibrium (SPE) for each stage. This calculation is carried out reverse to the order of occurrence and is aptly named backwards induction. Starting with the last stage, its decisions are informed by variables that have already been decided on and hence all previous decisions are taken as given. This then calculates an optimal decision relying on the previous stages. These previous stages are then solved in the same manner being informed by the decision patterns of upcoming decisions and taking preceding information as given.

The three stages of the model are:

1. The policy maker sets the policy instruments.
2. Each firm simultaneously decides on its investment into reductions in production costs and emissions.
3. Each firm simultaneously decides on the amount of output produced.

4.1. Third Stage: Production

The equilibrium of the third stage is achieved when each firm individually maximises its profits. If an internal solution exists the results of this maximisation are the individual outputs

$$q_i = \frac{p_0 + C + \tau_e E - (n+1)(c_i - \tau_e e_i)}{\eta(n+1)} = \frac{p_0 - c_i - \tau_e e_i}{\eta(n+1)}\,, \qquad \forall i \in \mathcal{F}\,; \qquad (9)$$

where $C = \sum_{i \in \mathcal{F}} c_i$ and $E = \sum_{i \in \mathcal{F}} e_i$ and the last equality holds only if firms are homogeneous.

Here, each firm takes into account the maximum price achievable on the market as well as their competition's costs subtracted by their costs and scaled by the price depreciation. This does not directly consider the investment made other than in the form of the costs and emissions they aim to reduce. The actual amount of money spent to achieve cost and emissions reductions is thus not important to the quantity produced. Further, a firm considers the emissions with the price it needs to pay for it.

Combining the individual outputs computes the total output

$$Q = \frac{n(p_0 - C - \tau_e E)}{\eta(n+1)}\,. \qquad (10)$$

These outputs fall in line with other last stages of Cournot models where the costs may be of a different functional form but the general result is in agreement. For comparison see Dorer (2022), Lambertini (2017), Lambertini et al. (2017) and Ulph and Ulph (2007).

To see that, if internal solutions to the Nash-equilibrium (NE) of the third stage exist, they are given by Equations (9) and (10), the first derivatives of profit with respect to output quantity need to vanish and the second derivatives need to be smaller than zero. Calculating the first order condition (FOC) by taking the derivative of profit yields the reaction functions for the firms:

$$\frac{\mathrm{d}\pi_i}{\mathrm{d}q_i} = p(Q) + p'(Q)q_i - c_i - \tau_e e_i \overset{!}{=} 0, \quad \forall i \in \mathcal{F}\,. \tag{11}$$

If this is an internal solution, all FOCs in Equation (11) are reaction functions that hold simultaneously.[6] Therefore, summing over all firms computes the relation between total output, price and unit costs as well as unit emissions in the third stage equilibrium:

$$ME = np(Q) + p'(Q)Q = C + \tau_e E = MC\,. \tag{12}$$

This equation shows that marginal earnings ME equal marginal costs MC, in case an internal solution exists.

Inserting the price function from Equation (1) and rearranging for Q yields Equation (10). This total output can in turn be inserted into the FOC together with the price function to calculate individual outputs as given in Equation (9), thus fulfilling the above.

4.2. Second Stage: Investment

Taking the result from the third stage, the NE of the second stage lends itself to the following proposition.

Proposition 1 *If the policymaker sets at least a carbon tax and an internal solution exists, a firm's marginal unit production costs and unit emissions costs are equal:*

$$\frac{dc_i}{dx_i^c} = \tau_e \frac{de_i}{dx_i^e}\,, \quad \forall i \in \mathcal{F}\,. \tag{13}$$

Further, for each firm, the total emissions and production costs are calculated by

$$q_i c_i = \frac{n+1}{2n\ell_i^c}(1+\tau_c)\,, \quad \forall i \in \mathcal{F}\,; \tag{14}$$

$$q_i \tau_e e_i = \frac{n+1}{2n\ell_i^e}(1+\tau_c)\,, \quad \forall i \in \mathcal{F}\,. \tag{15}$$

[6] The second order condition $p''(Q) + 2p'(Q) < 0$ is always fulfilled since the price is a linear decreasing function. Linear: $\forall n > 1 : p^{(n)}(Q) = 0$ and decreasing: $p'(Q) < 0$.

In case of perfect competition ($n = \infty$), the total production costs and emissions costs of a firm become

$$(q_i c_i)^\infty = \frac{1+\tau_c^\infty}{2\ell_i^c}\,, \quad \forall i \in \mathcal{F}\,; \tag{16}$$

$$(q_i \tau_e e_i)^\infty = \frac{1+\tau_c^\infty}{2\ell_i^e}\,, \quad \forall i \in \mathcal{F}\,. \tag{17}$$

In a state without an investment tax / subsidy, an increase in competition drives the firms to either lower their production value or costs to satisfy the shrinking term on the left-hand sides of Equations (14) and (15). This LHS turns into half of $(\ell_i^{c/e})^{-1}$ if there is perfect competition at $n = \infty$. When τ_c is present, its effects could negate or even inverse this strain on the firms to reduce their costs.

The equation of marginals indicates that the marginals of cost reductions need to be equal. Thus the last unit of money spent on production costs decreases yields the same decrease as the last unit of money spent on emissions costs decreases, indicating the most efficient allocation of funds. If a firm has high emissions with high abatement potential compared to a mature process with few benefits to be gained, it will invest in emissions until the benefit is equal to the production costs. On the other hand, if a production process involves a carbon-intensive process, that is hard to abate, the firm's main focus will be on reducing the production costs.[7]

Proof of Proposition 1: An internal solution to the profit maximisation with regards to the investments into emissions and production costs reduction, x_i^e and x_i^c, exists if the first derivative for investments is zero and the Hessian matrix of profit with respect to x_i^e and x_i^c is negative definite. Taking the first derivative of profit with respect to cost reduction investments and emissions reduction investments calculates the first-order conditions (FOCs).

$$-\frac{2n}{n+1} q_i \frac{\mathrm{d}c_i}{\mathrm{d}x_i^c} = (1+\tau_c)\,, \qquad \forall i \in \mathcal{F}\,; \tag{18a}$$

$$-\frac{2n}{n+1} \tau_e q_i \frac{\mathrm{d}e_i}{\mathrm{d}x_i^e} = (1+\tau_c)\,, \qquad \forall i \in \mathcal{F}\,; \tag{18b}$$

for which a detailed calculation is found in Appendix A.1.

These first-order conditions show that a firm maximises its profits by setting investments so that the marginal benefit from the investment equals the investment tax / subsidy. The policymaker can thus directly influence the firms' marginal benefit from investing and therefore the investment itself. The share of these investments spent on emissions abatement is however only controlled by τ_e.

[7] In theory, the model allows for negative investments, which could be understood as selling patents or equipment, but this should be avoidable through parameter choice.

Using the FOCs in Equations (18), Equation (13) is fulfilled when the RHSs of the two FOCs are equated with each other and simplified since the LHSs are equal. Computing the derivative of costs / emissions with regard to their respective investments from Equation (2) and inserting into Equations (A.4) yields Equations (14) and (15). Since $\lim_{\infty}(n+1)/n = 1$, Equations (16) and (17) that show the behaviour for perfect competition are also immediately calculated from Equations (14) and (15).

The calculation of the Hessian matrix of profits with regard to investments shows that it is negative definite if

$$(n+1)\eta q_i > n\left(c_i + \tau_e e_i\right) \,, \quad \forall i \in \mathcal{F} \,. \tag{19}$$

The detailed calculation is again shown in Appendix A.1. □

4.3. First Stage: Taxes and Subsidies

4.3.1. Central Symmetric Welfare as Benchmark

To have a comparison for the welfare achieved by setting the socially optimal taxes / subsidies for the decentralised market, I assume a policy maker who controls the firms' outputs and investments in a central manner. Since the actual economy consists of firms over whose decisions the policy maker has no influence, this is a hypothetical scenario.[8] I will refer to this setting as the centralised market in comparison to the decentralised market upon which the main analysis hinges.

The policymaker maximises all three variables simultaneously, neglecting the implications of the order of choices that form the three-stage game of the decentralised market. By definition, this hypothetical scenario yields the highest attainable welfare and thus introduces a good benchmark against which the attainable policies can be compared. Calculating the solution to this scenario yields the results

Proposition 2 *In the centralised market setting, the social optimum shows constant total costs from unit costs and emissions per firm:*

$$\alpha q_i e_i = \frac{1}{\ell_i^e} \qquad \forall i \in \mathcal{F} \,; \tag{20}$$

$$q_i c_i = \frac{1}{\ell_i^c} \qquad \forall i \in \mathcal{F} \,. \tag{21}$$

[8] If this scenario was possible a policy maker would most likely also have the ability to control the number of firms, inducing that there is only one firm, as this is most efficient. This would then be called a state-owned monopoly.

Further, the marginal unit costs are equal to the marginal unit damages

$$\frac{dc_i}{dx_i^c} = \alpha \frac{de_i}{dx_i^e}\,, \quad \forall i \in \mathcal{F}\,. \tag{22}$$

In the socially optimal scenario of the central market, the policy maker seeks firms to have costs equal to their inverse investment efficiency $(\ell_i^{c/e})^{-1}$ with no influence on the number of firms. Comparing these relations to the perfectly competitive case of the decentralised market in Equations (14) and (15) in the absence of an investment tax / subsidy, the costs of the perfectly competitive decentralised market are half that of the centralised market. Further, the comparison of Equation (13) with Equation (22) shows the only difference in the actual damages α being replaced by the fraction of damages the policy maker translates to the firms, represented by τ_e.

At first sight, it seems, that $\tau_e = \alpha$ and $\tau_c = (n-1)/(n+1)$ would make the system proposed in Proposition 1 equal to the system in Proposition 2, but the equation determining the output shows a difference. This makes the computation of the socially optimal taxes and subsidies slightly more complex, as the two following subsections will show.

Proof of Proposition 2: The social welfare function shows a local maximum in dependence on q_i, x_i^c and x_i^e whenever their derivatives vanish simultaneously and the Hessian matrix is zero at the point. Calculating the three derivatives and setting them to zero gives

$$\frac{\mathrm{d}U}{\mathrm{d}q_i} = \left(\frac{1}{2}(p_0 + p(Q)) - c_i - \alpha e_i\right) + \frac{1}{2}p'(Q)Q \overset{!}{=} 0, \qquad \forall i \in \mathcal{F}\,; \tag{23a}$$
$$\frac{\mathrm{d}U}{\mathrm{d}x_i^c} = -q_i \frac{\mathrm{d}c_i}{\mathrm{d}x_i^c} - 1 \overset{!}{=} 0, \qquad \forall i \in \mathcal{F}\,; \tag{23b}$$
$$\frac{\mathrm{d}U}{\mathrm{d}x_i^e} = -\alpha q_i \frac{\mathrm{d}e_i}{\mathrm{d}x_i^e} - 1 \overset{!}{=} 0, \qquad \forall i \in \mathcal{F}\,. \tag{23c}$$

Taking Equation (23b) and Equation (23c) and using the investment function to compute the derivatives of production costs and emission costs gives Equations (20) and (21) by rearrangement. The SOCs namely the Hessian of the profit being negative definite is satisfied whenever

$$\eta q_i > c_i + \alpha e_i\,. \tag{24}$$

The equation of marginals in (22) is computed by equating Equations (23b) and (23c). $\square$

4.3.2. Emissions Tax

Considering the firms' decisions on investments leads to the last stage of the calculation and the first stage of the game: setting the taxes. Beginning with the carbon tax shows that

Proposition 3 *For any choice of investment tax / subsidy τ_c the equation*

$$\tau_e = \frac{n+1}{2n}(1+\tau_c)\alpha \tag{25}$$

denotes the socially optimal carbon tax. This tax implies constant total emissions per firm for the decentralised market.

The policymaker directly takes into account not only the number of firms but also whether the investment tax / subsidy exists or not. In the latter case, the carbon tax partially acts as a substitute for τ_c. This becomes apparent when once more looking at the case of perfect competition and comparing it to the monopoly. The perfect competition suggests that the carbon tax should be $\alpha/2$ while the socially optimal carbon tax for the monopoly is α. Following the standard Pigouvian formulation of a carbon tax, it should be α in the case of perfect competition and not for a monopoly. Therefore the assumption seems reasonable that τ_e corrects for more than the damages alone.

Proof of Proposition 3 The socially optimal taxes are again given by inserting the solution to the second stage into the social welfare function and taking the derivative with respect to taxes. Examining the dependence of output on the carbon tax as given in Equation (9) under the insertion of Equation (13) leaves no dependence on τ_e. With this information the solution to the implicit equation given in Equation (14) reveals that

$$\frac{\mathrm{d}c_i}{\mathrm{d}\tau_e} = 0\,, \qquad \forall i \in \mathcal{F}\,; \tag{26}$$

as the derivative of the implicit equation with respect to τ_e is 0 and the implicit function theorem reveals that the derivative is therefore also zero.

Thus the only terms with dependency on τ_e are the emissions per unit and the investment into emissions reduction. Taking their derivatives and inserting them into the derivative of the social welfare function for carbon taxes calculates

$$\frac{\mathrm{d}U}{\mathrm{d}\tau_e} = \sum_{i\in\mathcal{F}} \left[-\alpha q_i \frac{\mathrm{d}e_i}{\mathrm{d}\tau_e} - \frac{\mathrm{d}x_i^e}{\mathrm{d}\tau_e}\right] = \left[\alpha\frac{n+1}{2n\tau_e^2}(1+\tau_c) - \frac{1}{\tau_e}\right]\sum_{i\in\mathcal{F}}\frac{1}{\ell_i^e} \stackrel{!}{=} 0 \tag{27}$$

Reformulating Equation (27) under the assumption that all ℓ_i^e are finite and defined, yields Equation (25). Inserting this equation into Equation (15) yields $\alpha q_i \ell_i^e e_i = 1$, confirming the second statement and thus all of Proposition 3. □

4.3.3. Investment Tax / Subsidy

The complexity of the investment tax / subsidy allows a well-contained solution only for the case of perfect competition. As seen in Appendix A.2.3, the equation from which τ_c is calculated, is a polynomial of third order and thus analytically solvable. However, the complexity of its results defies interpretation. Therefore, the full discussion of this policy tool is relegated to the numerical illustration in the next part.

Proposition 4 *The investment tax / subsidy is $\tau_c = 1$ for perfectly competitive markets ($n = \infty$). With this investment tax the carbon tax becomes $\tau_e = \alpha$ in a perfectly competitive market.*

This result confirms that perfect competition diminishes the market power effects present in an oligopoly. In case of perfect competition, a policymaker sets the carbon tax equal to the damages that a unit of carbon causes. Further, fully competitive firms seem to engage in strategic over-investment where the individual firms invest too much, thus driving up costs for every competitor as well.

Proof of Proposition 4: The investment tax / subsidy solves the equation shown in Equation (A.22). The highest two orders of n present in this formula have the coefficients shown in Equations (28) and (29)

$$b_3 = \ell_i^c(\tau_c - 1)^2 + \frac{(\ell_i^c - \ell_i^e)(\ell_i^c + \ell_i^e)(\tau_c - 1)(\ell_i^c(2\tau_c - 1) - \ell_i^e(2\tau_c + 1))\eta}{(\ell_i^c)^2 \ell_i^e p_0^2} \tag{28}$$

$$b_4 = \frac{(\ell_i^c - \ell_i^e)^2(\ell_i^c + \ell_i^e)(\tau_c - 1)^2\eta}{2(\ell_i^c)^2 \ell_i^e p_0} \tag{29}$$

Supposing that τ_c is a solution to the underlying equation, in the case of $n \to \infty$, the term of highest order needs to vanish individually. Hence $\tau_c = 1$ is the solution to this equation for a perfectly competitive market if $\ell_i^c \neq \ell_i^e$.

If the investment efficiencies are indeed similar ($\ell_i^c = \ell_i^e$), the highest order coefficient vanishes, leaving only the first term of the second highest order coefficient in Equation (28). However, this coefficient still implies that $\tau_c = 1$ and hence the investment tax / subsidy is 1 for perfectly competitive markets.

With the investment tax calculated, taking Equation (25) and inserting the value for the carbon tax gives

$$\begin{aligned} \tau_e &= \frac{n+1}{2n} \cdot 2\alpha \\ \Rightarrow \lim_{n\to\infty} \tau_e &= \lim_{n\to\infty} \frac{n+1}{n}\alpha = \alpha \end{aligned} \tag{30}$$

which shows what was stated. □

5. Illustration

As the analytical calculation is not complete in its solutions and interpretations I aid it through numerical illustrations. From Proposition 1 and the results to the third stage, explicit formulas for c_i, e_i and q_i can be derived:

$$c_i = \frac{1}{2\ell_i^c m_\ell}\left[p_0 - \sqrt{p_0^2 - \frac{2m_\ell(1+\tau_c)\eta(n+1)^2}{n}}\right], \qquad \forall i \in \mathcal{F}; \tag{31}$$

$$\tau_e e_i = \frac{\ell^c}{\ell^e}c_i = \frac{1}{2\ell^e m_\ell}\left[p_0 - \sqrt{p_0^2 - \frac{2m_\ell(1+\tau_c)\eta(n+1)^2}{n}}\right], \qquad \forall i \in \mathcal{F}; \tag{32}$$

$$q_i = \frac{p_0 - c_i - \tau_e e_i}{\eta(n+1)} = \frac{1}{2\eta(n+1)}\left[p_0 + \sqrt{p_0^2 - \frac{2m_\ell(1+\tau_c)\eta(n+1)^2}{n}}\right], \quad \forall i \in \mathcal{F}; \tag{33}$$

where $m_\ell = (1/\ell_i^c + 1/\ell_i^e)$ is the sum of the inverse investment efficiencies. From Equations (31) and (32), $x_i^{c/e}$ can be derived by dividing through $(c/e)_i^0$, taking the logarithm and dividing by $\ell_i^{c/e}$, thereby inverting Equations (2) and (3).

These equations are still dependent on the investment tax / subsidy since it is not solved by a compact analytical solution and I thus calculate it numerically. All illustrations on display are for fully homogeneous firms. The code for this and all further calculations are included in the online appendix.

5.1. Defining Scenarios

To best highlight the results of this small model, I define four scenarios to illustrate the different qualities. The parameters to these scenarios are given in Table 1. The first scenario is the *Symmetric* scenario, where all firms have theoretically completely symmetric cost structures in unit emissions and unit costs. In formulas: $\alpha = 1$ and $\ell_i^c = \ell_i^e$ as well as $c_i^0 = e_i^0$. The second and third scenarios explore the difference between unit emissions and unit costs by setting one of them higher and harder to reduce. These are called the *Higher Emissions* and the *Higher Costs* scenario where the high and hard to abate value h and the low value l are related as $h_i^0 = 2l_i^0$ and $2\ell_i^h = \ell_i^l$, while all other parameters are kept equal to the first scenario. The last scenario is the *High Damage* scenario that explores the influence of a doubling in damages from emissions.

Set Name	p_0	η	α	c^0	ℓ^c	e^0	ℓ^e
Symmetric	30.0	0.5	1.0	4.0	0.6	4.0	0.6
High Emissions	30.0	0.5	1.0	4.0	0.6	8.0	0.3
High Costs	30.0	0.5	1.0	8.0	0.3	4.0	0.6
High Damages	30.0	0.5	2.0	4.0	0.6	4.0	0.6

Table 1: Parameters used for the numerical illustrations.

5.2. Policy Values and Welfare

Solving the equation for the investment tax / subsidy numerically for the four market scenarios shows that in every scenario τ_c is a subsidy for a monopoly and oligopoly but then crosses the zero-line to become a tax, decreasing the firms' incentive to invest as seen in Figure 1. While the different scenarios slightly move the start point of the curve for τ_c, the shape remains the same for all four scenarios. As stated in Proposition 4, all scenarios approach the limit value of $\tau_c^\infty = 1$.

The emissions tax is always a tax. It starts in the region of about half the damages from emissions. If investments into emissions reductions are hard to achieve as in the *High Emissions* scenario, it starts lower and higher for easier abatement as in *High Costs* scenario. The relationship between τ_e and α is fully linear and thus both of their curves look identical. It tends to α as $n \to \infty$. This is opposite to the behaviour observed in the absence of τ_c as explained in the previous section, thus the ideal τ_c fully reverses the effect described there.

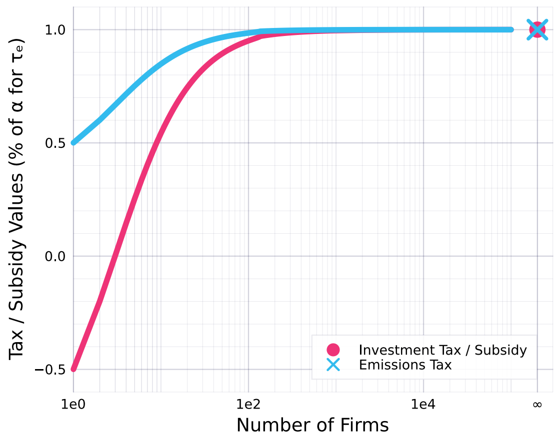

Figure 1: This figure shows the carbon and investment policy for the *Symmetric* scenario of Table 1. The chart has a logarithmic scale to highlight the progression of the curves and values shown for the emissions tax corresponding to τ_e/α.

These two behaviours show that the firms in the beginning especially in the case of a monopoly under-supply the market with goods and it is in the interest of the policy maker to have artificially low prices so that firms' production costs decrease thus increasing their incentive to produce. Once competition stiffens, the incentive to invest increases along with the incentive to fully supply the market, rendering this correction unnecessary. From there on the policymaker only uses the two policies as taxes to correct for over-investment (τ_c) and the emissions externality (τ_e). If any, the differences in α seem to have a minute effect on the shape or position of the policy curves.

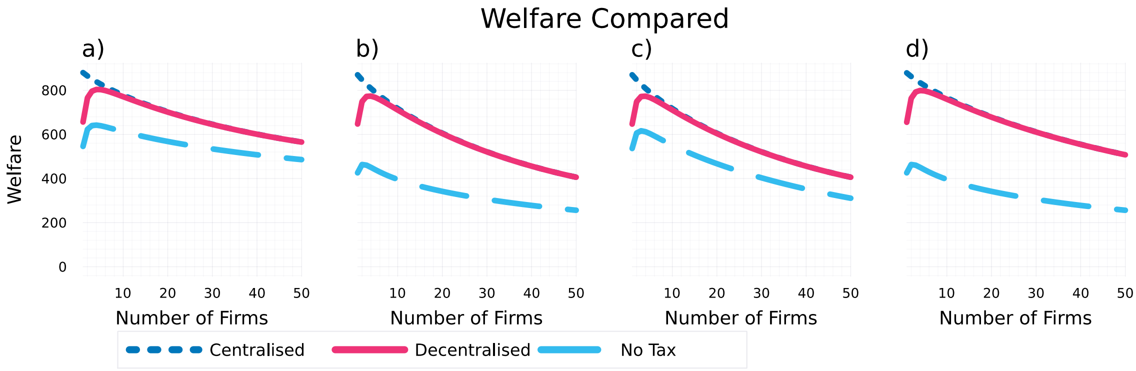

Figure 2: The above shows welfare for the model compared to the centralised market. Panel a) to d) refer to scenarios *Symmetric*, *High Emissions*, *High Costs* and *High Damages* respectively. The *No Tax* keyword refers to a scenario with no taxation.

The plot on welfare in Figure 2 shows a clear hierarchy in welfare for all four scenarios presented in Table 1. The decentralised market with guidance from a policymaker is always superior to the market without guidance. The decentralised market with policy guidance quickly tends towards the centralised benchmark scenario as the number of firms increases. Since the maximum price, the consumers are willing to pay, p_0, and the price depreciation with quantity, η, are equal, the initial welfare for the monopoly is comparable across scenarios, actually allowing for this comparison.

The *Symmetric* and *High Costs* scenarios are easily distinguished from the *High Emissions* and *High Damages* scenarios, for the latter two show lower welfare in the setting without a policy maker, indicating higher climate damages. The two scenarios with *High Emissions* and *High Costs* show the steepest decrease in welfare, because higher total investment costs diminish the firms' profits without an obvious increase in consumer surplus. Lastly, the rate of decrease of total welfare is higher in the setting with a policy maker, having the respective curve approach that of the setting without a policy maker for increasing competition.

5.3. Investments, Profits and Outputs

Figure 3 shows that in the *High Emissions* and *High Costs* scenario, firms in the decentralised market invest slightly more in cost reductions and slightly less in emissions reductions than would be recommended by the centralised market. This behaviour is observed for all four scenarios. Further, the investments follow the curve of an inverted-U relationship, while the per-firm investments form a continuously decreasing curve of decreasing slope.

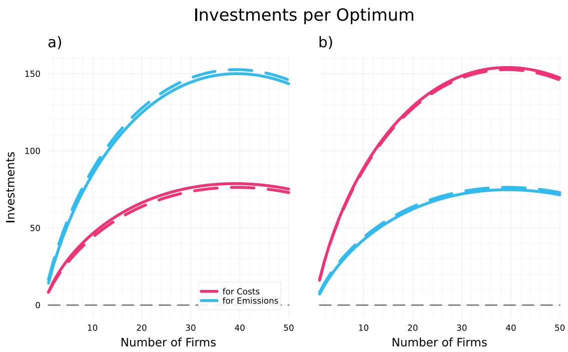

Figure 3: This figure shows the investments of all firms a) *High Emissions* and b) *High Costs*. The solid lines refer to the decentralised market setting while the dashed line shows the corresponding benchmark from the centralised market setting.

Figure 4 breaks down the profits of an individual firm by way of an example. The coloured curves indicate the composition of profits: Starting with the red line, it shows the earnings from selling the goods on the market, the orange-shaded region is the amount the earnings decrease on behalf of investments while the pink region is the production costs paid. The blue line can then be interpreted as profit before taxes or after tax return, as the policymaker's budget is balanced. The light blue then indicates carbon taxes and the dark blue investment taxes paid. Therefore, the green line is the profit of the firms before taxes have been refunded.

The grey lines indicate the profits of a market of non-investing firms for the dotted line and a market of non-producing firms for the dashed line. Firms profits fall below that of the market without investments when for more than 3 firms competing in the market and below the non-producing at around $N = 16$. The decision whether firms invest in the first place is never asked in the model and thus always answered with yes.

Looking at the firms' individual output in Figure 5, shows that the firms of the decentralised market still take advantage of their position in the market by under-supplying goods to get the price up. In the monopoly, the decentralised market - regulated or not - shows only half the output of the centralised market. Looking at the total output in the second panel, the output only diverges from its unregulated counterpart once its values approach the ones of the centralised market, whereas the unregulated market passes the centralised market in its total output. As expected from Proposition 2 the centralised market has constant per firm total production. The output of the unregulated market starts at the same value but quickly falls to half that of the centralised market as was also discussed in the previous section. The previously described behaviour is inverted by the investment tax / subsidy comparable to the emissions tax where the regulated decentralised market starts at half of the value of the centralised market but approaches $1/\ell_i^c$ as $n \to \infty$. These initial deviations of the outputs are not as apparent in the last panel giving the impression of three straight lines, while the only true straight line is the one of the centralised market. Costs from emissions for the individual firm form two nearly constant lines for

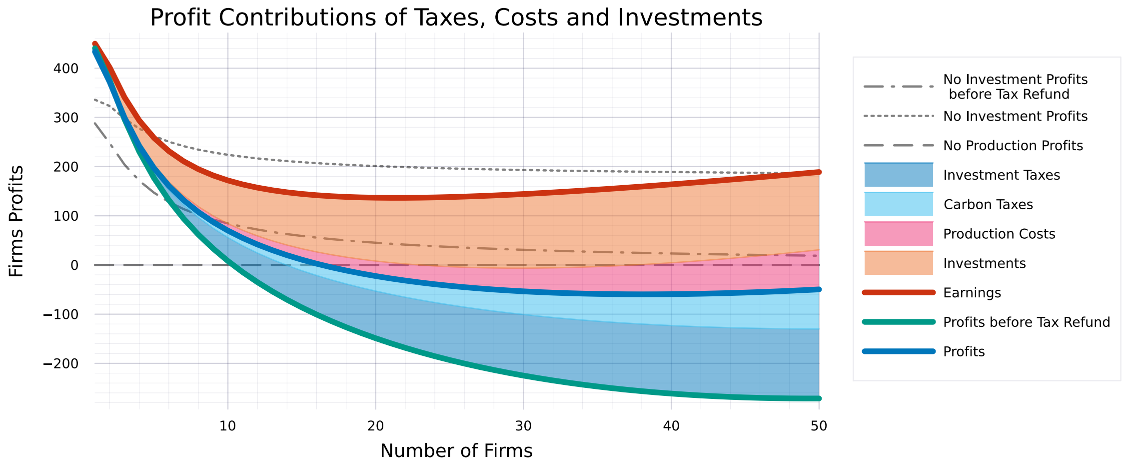

Figure 4: The firms profit in the *Symmetric* case. The red curve shows the earnings of the firm, the green curve shows the profit before taxes are refunded by the policy maker and the blue line shows the profits after taxes have been refunded. The reddish shadings can be considered the firms' expenses while the blue shadings are tax refunds. The grey lines show the profits if firms were to not invest in cost / emissions reductions before and after tax refunds, as well as if they would not produce at all.

the centralised and decentralised market with guidance. This is likely explained by the emissions tax quickly approaching the actual damages. The emissions of the unregulated market can be inferred in their shape from the total and individual output respectively since the firms see no incentive to invest in emissions reduction without taxation.

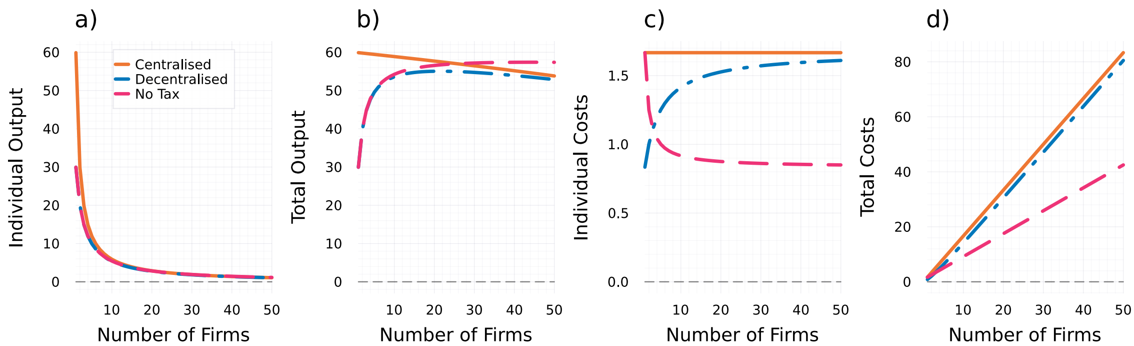

Figure 5: Outputs and costs per firm as well as all of society. The costs refer to the total accruing costs in both cases, not unit costs. All four figures are generated with the *Symmetric* scenario. a) refers to the per firm output, b) is the output of the whole economy, c) is the total production costs an individual firm faces, while d) shows the total production costs the whole economy faces.

6. Discussion and Conclusion

The model explored in this work confirms previous research on Cournot models in the field of green innovation (Dorer, 2022; Lambertini et al., 2017; Ulph & Ulph, 2007). It is a complex model displaying the following three inefficiencies: market power, over-investment and the externality of climate change. The policies in place have the power to change the behaviour of the firms but cannot fully address the inefficiency of market power. Therefore, it is only in the special case of a perfectly competitive market, that the policymaker can address all inefficiencies. However, the main finding is that the policymaker can influence the investment decisions of a single firm without an explicit handle on green investments. In fact, firms abate their emissions as soon as they face costs for emitting carbon and the firms' emissions are close to those of the centralised market.

In more detail, the two policies proposed - as long as both are exerted - are in agreement with research on the topic. Suppose the carbon tax is adopted in combination with a policy steering investments. In that case, it is between half of the cost of carbon and the full cost in accordance with (Elkins & Baker, 2001) who quote often lower than price carbon taxes still being effective where observed. However, if the carbon tax is the only policy measure put in place to address the inefficiencies of the market, the policymaker would want to weigh the effect of the carbon tax against its effect on investment decisions, thereby lowering it from the optimal level to steer the industry away from over-investment for high numbers of firms in the market.

On a further note, while not having the same functional shape as the one proposed by Lambertini et al. (2017), I can report on an inverted-U relationship between investment and competition. This result is rather unexpected since the authors attributed these results to spillovers and did not observe it in case of no spillovers, which is contradicting my results. Further, I can also observe the strategic over-investment shown by Ulph and Ulph (2007).

There are a few facts to consider: First the investment policy is a tax in the majority of cases rather than a subsidy. Inside the model, this can easily be attributed to the *strategic over-investment* described before by Ulph and Ulph (2007) who report similar results. This is not really observed in the "real world". Indeed, there are multiple lines of evidence that point towards the problem of firms investing too little instead of too much (Becker, 2015; Reichenbach & Requate, 2012). I will highlight two possible explanatory avenues for this result.

One aspect that has not been considered here is the fact, argued for by Schumpeter (1942), that investments into innovation always involve a risk for the business. Since most market participants are considered risk-averse rather than risk-taking, this would decrease the likelihood of research or increase its costs. This is attributed to mechanisms like the risk premium but also liquidity constraints required to keep businesses operational (Pratt, 1964).

A second aspect of research mentioned in the literature review but not included in the model is the subject of spillovers (Dorer, 2022; Fischer & Newell, 2008; Reichenbach & Requate, 2012; Verspagen, 1997). They work in two ways: First, they (partly) diffuse the knowledge retrieved by one firm to all others thereby making investments more valuable to a policymaker since diffused innovation increases societal rather than individual knowledge. Second, as a firm strives for competitive advantage, the spillover makes the investment less valuable for a firm. Both effects should work towards making investments more instead of less desirable from the policymaker's perspective.

Even though the model is in this regard not appreciative of the real world, the mechanism exhibited should be kept in mind when looking at the idea of perfect competition. An example could be two firms spending a lot of money to find the same technique to produce better photovoltaic panels. While both firms profit from their novel photovoltaic panel, the panels are more expensive since they need to pay for two times the research effort. In the "real world" information flows more freely, be it through industrial espionage or academic research, but to some extent, the effect might still be observable.

Another issue that becomes apparent in the discussion of Figure 4, is the lack of a clear decision scheme on firms' investments. While this model just assumed that firms are always willing to invest, the real world could for example better be modelled by a coalition game where only a fraction of firms invests and the rest simply produces with smaller profits. This has not been addressed in my model due to time constraints and would be an interesting further path for future research.

Combining the thoughts on coalition formation with the mechanisms of spillovers and market power, I think that the model could provide interesting insights into cooperation. Cooperation can take many shapes in the model: the cooperative decision to stop investing instead of playing a game of chicken, the cooperation in research to reduce individual research costs, spillovers which could be understood as unwanted research cooperation or even market collusion, cooperating to maximise profits as if being a monopoly. Each of these would pose an interesting case to investigate.

Another aspect that is missing in the formulation of this model, is a time component. Climate change itself is not a problem of instantaneous but of long-lived nature. When the question of how to set a rate for the discount on future generations' welfare has sparked a huge debate (Nordhaus, 2007; Stern, 2007; Sterner & Persson, 2008; Weitzman, 2007), it seems negligent to ignore the temporal component. Even more so, when considering that investments also bear a temporal component, that justifies their extent over time and is able to offset some of the previous expenses. Along with the inclusion of spillover effects, this might be an avenue for further research.

Finally the functional forms of the investment curves lack any justification beyond satisfying the constraints posed by the model, even though their inclusion was argued for on behalf of learning curves. The same holds true for the linear inverse demand function and the linear damage curve which have been chosen for simplicity as well. Their effects on the outcome of this modelling seem substantial and have not been explored.

7. Conclusion

Whether the results would be jeopardised by a model more appreciative of the real world is unclear at this stage and a question which cannot be answered at here anymore. Nevertheless in the proposed and analysed model, a policy maker with no direct influence on the investments into emissions reduction investments can steer the economy onto an emissions path that is equal to the one of a centralised economy. While market power effects cannot be fully addressed by the suite of policies available to the policy maker, they achieve a social welfare that tends towards the social welfare of a centralised market for increasing competition. Even though this is a highly stylised and flawed model, it leaves me curious whether these results have at least some validity in the real world.

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A. Further Calculations

These are the calculations to support the surface level calculations of Section 4

A.1. Second Stage

This part highlights the calculation of Equations (A.4). The derivative with respect to costs investments and emissions investments are computationally equivalent except for the factor of τ_e. Therefore, the calculation below shows the derivation with respect to emissions reduction investments, the costs reduction investments follow the same principles. Taking the first derivative of profit with respect to investment gives

$$\frac{\mathrm{d}\pi}{\mathrm{d}x_i^e} = (1-\tau_q)\left[p'(Q)q_i\frac{\mathrm{d}Q}{\mathrm{d}x_i^e} - \tau_e q_i \frac{\mathrm{d}e_i}{\mathrm{d}x_i^e} + (p(Q) - c_i - \tau_e e_i)\frac{\mathrm{d}q_i}{\mathrm{d}x_i^e}\right] - (1+\tau_c) = 0 \tag{A.1}$$

Here the derivatives of output with respect to investments are to be determined. To compute them Equation (3) is used for the derivative of emissions with respect to investments. To then find estimates for the derivatives of the individual and total output with respect to emissions, we take the total differential of Equations (11) and (12):

$$\begin{aligned} p'(Q)\mathrm{d}Q + p''(Q)\mathrm{d}Q + p'(Q)\mathrm{d}q_i &= \mathrm{d}c_i + \tau_e \mathrm{d}e_i\,, \\ np'(Q)\mathrm{d}Q + p''(Q)\mathrm{d}Q + p'(Q)\mathrm{d}Q &= \mathrm{d}C + \tau_e\mathrm{d}E = \sum_{i\in\mathcal{F}}(\mathrm{d}c_i + \tau_e\mathrm{d}e_i)\;. \end{aligned}$$

In the above the terms containing a second derivative of price vanish, since price is a linear function. Varying either $\mathrm{d}c_i$ or $\mathrm{d}e_i$, keeping everything else constant, which especially states that $\forall i \neq j : \mathrm{d}x_i/\mathrm{d}y_j = 0$ where x and y can be substituted by c and e, gives the change of total output with respect to costs or emissions:

$$\begin{aligned} (n+1)p'(Q)\mathrm{d}Q &= \sum_{i\in\mathcal{F}}(\mathrm{d}c_i + \tau_e\mathrm{d}e_i)\;, \\ \Rightarrow \quad \frac{\mathrm{d}Q}{\mathrm{d}e_i} &= \frac{\tau_e}{(n+1)p'(Q)}\,. \end{aligned} \tag{A.2}$$

which allows the calculations of change in individual output to

$$\begin{aligned} p'(Q)(\mathrm{d}q_i + dQ) &= \mathrm{d}c_i + \tau_e\mathrm{d}e_i\,, \\ \Rightarrow \quad \frac{\mathrm{d}q_i}{\mathrm{d}e_i} &= \frac{\tau_e}{p'(Q)} - \frac{\mathrm{d}Q}{\mathrm{d}e_i} = \frac{n\cdot\tau_e}{(n+1)p'(Q)}\,. \end{aligned} \tag{A.3}$$

With the computations of the derivatives of output with respect to investments done, they can be inserted into Equation (A.1).

$$\begin{aligned}
\left[\cancel{p'(Q)}q_i\frac{\tau_e}{(n+1)\cancel{p'(Q)}}-\tau_e q_i-\cancel{p'(Q)}q_i\frac{n\cdot\tau_e}{(n+1)\cancel{p'(Q)}}\right]\frac{\mathrm{d}e_i}{\mathrm{d}x_i^e} &= (1+\tau_c)\,,\\
\frac{1-n-1-n}{n+1}\tau_e q_i\frac{\mathrm{d}e_i}{\mathrm{d}x_i^e} &= (1+\tau_c)\,,\\
-\frac{2n}{n+1}\tau_e q_i\frac{\mathrm{d}e_i}{\mathrm{d}x_i^e} &= (1+\tau_c)\,.
\end{aligned}$$

Which then gives the two first order conditions for this stage, from which the equations shown in Equations (A.4) can be calculated by rearrangement.

$$\frac{\mathrm{d}\pi_i}{\mathrm{d}x_i^c}=\frac{2n}{n+1}q_i\frac{\mathrm{d}c_i}{\mathrm{d}x_i^c}+(1+\tau_c)\overset{!}{=}0\,,\qquad \forall i\in\mathcal{F}\,; \tag{A.4a}$$

$$\frac{\mathrm{d}\pi_i}{\mathrm{d}x_i^e}=\frac{2n}{n+1}\tau_e q_i\frac{\mathrm{d}e_i}{\mathrm{d}x_i^e}+(1+\tau_c)\overset{!}{=}0\,.\qquad \forall i\in\mathcal{F}\,; \tag{A.4b}$$

Now, to ensure that this first stage is also trying to find a maximum, calculate the four second derivatives:

$$\frac{\mathrm{d}^2\pi_i}{\left(\mathrm{d}x_i^c\right)^2}=-\frac{2n}{n+1}\left[\frac{\mathrm{d}q_i}{\mathrm{d}x_i^c}\frac{\mathrm{d}c_i}{\mathrm{d}x_i^c}+q\frac{\mathrm{d}^2c_i}{\left(\mathrm{d}x_i^c\right)^2}\right] \tag{A.5a}$$

$$\frac{\mathrm{d}^2\pi_i}{\mathrm{d}x_i^c\partial x_i^e}=-\frac{2n}{n+1}\left[\frac{\mathrm{d}q_i}{\mathrm{d}x_i^e}\frac{\mathrm{d}c_i}{\mathrm{d}x_i^c}+q_i\frac{\mathrm{d}^2c_i}{\mathrm{d}x_i^c\partial x_i^e}\right] \tag{A.5b}$$

$$\frac{\mathrm{d}^2\pi_i}{\mathrm{d}x_i^e\partial x_i^c}=-\frac{2n\cdot\tau_e}{n+1}\left[\frac{\mathrm{d}q_i}{\mathrm{d}x_i^c}\frac{\mathrm{d}e_i}{\mathrm{d}x_i^e}+q_i\frac{\mathrm{d}^2e_i}{\mathrm{d}x_i^e\partial x_i^c}\right] \tag{A.5c}$$

$$\frac{\mathrm{d}^2\pi_i}{\left(\mathrm{d}x_i^e\right)^2}=-\frac{2n\cdot\tau_e}{n+1}\left[\frac{\mathrm{d}q_i}{\mathrm{d}x_i^e}\frac{\mathrm{d}e_i}{\mathrm{d}x_i^e}+q\frac{\mathrm{d}^2e_i}{\left(\mathrm{d}x_i^e\right)^2}\right] \tag{A.5d}$$

In Equations (A.5) $\frac{\mathrm{d}^2c}{\mathrm{d}x^c\partial x^e}=0$ as well as $\frac{\mathrm{d}^2e}{\mathrm{d}x^e\partial x^c}=0$. Then it can be rewritten as a Hessian matrix

$$H_\pi=-\frac{2n}{n+1}\begin{pmatrix}\left[\frac{\mathrm{d}q_i}{\mathrm{d}x_i^c}\frac{\mathrm{d}c_i}{\mathrm{d}x_i^c}+q_i\frac{\mathrm{d}^2c_i}{\left(\mathrm{d}x_i^c\right)^2}\right] & \frac{\mathrm{d}q_i}{\mathrm{d}x_i^e}\frac{\mathrm{d}c_i}{\mathrm{d}x_i^c}\\ \tau_e\frac{\mathrm{d}q_i}{\mathrm{d}x_i^c}\frac{\mathrm{d}e_i}{\mathrm{d}x_i^e} & \tau_e\left[\frac{\mathrm{d}q_i}{\mathrm{d}x_i^e}\frac{\mathrm{d}e_i}{\mathrm{d}x_i^e}+q_i\frac{\mathrm{d}^2e_i}{\left(\mathrm{d}x_i^e\right)^2}\right]\end{pmatrix}$$

If its Eigenvalues are negative the calculated point is a maximum. Since $\det H_\pi=\lambda_1\cdot\lambda_2$, if the Determinant of the Hessian is positive and one of the diagonal elements is negative, this is given. Start with showing its positive determinant:

$$
\begin{aligned}
\det H_\pi &= \frac{4n^2\tau_e}{(n+1)^2}\left(\left[\frac{\mathrm{d}q_i}{\mathrm{d}x_i^c}\frac{\mathrm{d}c_i}{\mathrm{d}x_i^c} + q_i\frac{\mathrm{d}^2 c_i}{\left(\mathrm{d}x_i^c\right)^2}\right]\cdot\left[\frac{\mathrm{d}q_i}{\mathrm{d}x_i^e}\frac{\mathrm{d}e_i}{\mathrm{d}x_i^e} + q_i\frac{\mathrm{d}^2 e_i}{\left(\mathrm{d}x_i^e\right)^2}\right] - \frac{\mathrm{d}q_i}{\mathrm{d}x_i^e}\frac{\mathrm{d}c_i}{\mathrm{d}x_i^c}\frac{\mathrm{d}q_i}{\mathrm{d}x_i^c}\frac{\mathrm{d}e_i}{\mathrm{d}x_i^e}\right)\\
&= \frac{4n^2\tau_e}{(n+1)^2}\left(\cancel{\frac{\mathrm{d}q_i}{\mathrm{d}x_i^e}\frac{\mathrm{d}e_i}{\mathrm{d}x_i^e}\frac{\mathrm{d}q_i}{\mathrm{d}x_i^c}\frac{\mathrm{d}c_i}{\mathrm{d}x_i^c}} + \frac{\mathrm{d}q_i}{\mathrm{d}x_i^c}\frac{\mathrm{d}c_i}{\mathrm{d}x_i^c}q_i\frac{\mathrm{d}^2 e_i}{\left(\mathrm{d}x_i^e\right)^2} + q_i\frac{\mathrm{d}^2 c_i}{\left(\mathrm{d}x_i^c\right)^2}\frac{\mathrm{d}q_i}{\mathrm{d}x_i^e}\frac{\mathrm{d}e_i}{\mathrm{d}x_i^e} + \right.\\
&\qquad\left. + q_i^2\frac{\mathrm{d}^2 c_i}{\left(\mathrm{d}x_i^c\right)^2}\frac{\mathrm{d}^2 e_i}{\left(\mathrm{d}x_i^e\right)^2} - \cancel{\frac{\mathrm{d}q_i}{\mathrm{d}x_i^e}\frac{\mathrm{d}c_i}{\mathrm{d}x_i^c}\frac{\mathrm{d}q_i}{\mathrm{d}x_i^c}\frac{\mathrm{d}e_i}{\mathrm{d}x_i^e}}\right)\\
&= \frac{4n^2\tau_e q_i}{(n+1)^2}\left(\frac{\mathrm{d}q_i}{\mathrm{d}c_i}\left(\frac{\mathrm{d}c_i}{\mathrm{d}x_i^c}\right)^2\frac{\mathrm{d}^2 e_i}{\left(\mathrm{d}x_i^e\right)^2} + \frac{\mathrm{d}^2 c_i}{\left(\mathrm{d}x_i^c\right)^2}\frac{\mathrm{d}q_i}{\mathrm{d}e_i}\left(\frac{\mathrm{d}e_i}{\mathrm{d}x_i^e}\right)^2 + q_i\frac{\mathrm{d}^2 c_i}{\left(\mathrm{d}x_i^c\right)^2}\frac{\mathrm{d}^2 e_i}{\left(\mathrm{d}x_i^e\right)^2}\right)
\end{aligned}
$$

We strive to proof that $\det H_\pi > 0$, so restricting the result to $q_i \neq 0$ this gives

$$
\begin{aligned}
&\cancel{\frac{4n^2\tau_e q_i}{(n+1)^2}}\left(\frac{\mathrm{d}q_i}{\mathrm{d}c_i}\left(\frac{\mathrm{d}c_i}{\mathrm{d}x_i^c}\right)^2\frac{\mathrm{d}^2 e_i}{\left(\mathrm{d}x_i^e\right)^2} + \frac{\mathrm{d}^2 c_i}{\left(\mathrm{d}x_i^c\right)^2}\frac{\mathrm{d}q_i}{\mathrm{d}e_i}\left(\frac{\mathrm{d}e_i}{\mathrm{d}x_i^e}\right)^2 + q_i\frac{\mathrm{d}^2 c_i}{\left(\mathrm{d}x_i^c\right)^2}\frac{\mathrm{d}^2 e_i}{\left(\mathrm{d}x_i^e\right)^2}\right) > 0\\
\Leftrightarrow\qquad & q_i\frac{\mathrm{d}^2 c_i}{\left(\mathrm{d}x_i^c\right)^2}\frac{\mathrm{d}^2 e_i}{\left(\mathrm{d}x_i^e\right)^2} > -\left(\frac{\mathrm{d}q_i}{\mathrm{d}c_i}\left(\frac{\mathrm{d}c_i}{\mathrm{d}x_i^c}\right)^2\frac{\mathrm{d}^2 e_i}{\left(\mathrm{d}x_i^e\right)^2} + \frac{\mathrm{d}^2 c_i}{\left(\mathrm{d}x_i^c\right)^2}\frac{\mathrm{d}q_i}{\mathrm{d}e_i}\left(\frac{\mathrm{d}e_i}{\mathrm{d}x_i^e}\right)^2\right).
\end{aligned}
$$

Inserting the derivatives displayed, gives

$$
\begin{aligned}
q_i\cancel{(\ell_i^c\ell_i^e)^2}c_i e_i &> \frac{n}{(n+1)\eta}\cancel{(\ell_i^c\ell_i^e)^2}\cdot\left(c_i^2 e_i + \tau_e c_i e_i^2\right)\\
q_i\cancel{c_i}\cancel{e_i} &> \frac{n}{(n+1)\eta}\left(c_i^{\cancel{2}}\cancel{e_i} + \tau_e\cancel{c_i}e_i^{\cancel{2}}\right)\\
(n+1)\eta q_i &> n(c_i + \tau_e e_i)
\end{aligned}
$$

which is displayed in Equation (19).

Focusing on the diagonal elements being negative

$$
\begin{aligned}
-\frac{2n}{n+1}\left[\frac{\mathrm{d}q_i}{\mathrm{d}x_i^c}\frac{\mathrm{d}c_i}{\mathrm{d}x_i^c} + q_i\frac{\mathrm{d}^2 q_i}{\left(\mathrm{d}x_i^c\right)^2}\right] &> 0\\
\frac{\mathrm{d}q_i}{\mathrm{d}x_i^c}\frac{\mathrm{d}c_i}{\mathrm{d}x_i^c} &> -q_i\frac{\mathrm{d}^2 q_i}{\left(\mathrm{d}x_i^c\right)^2}
\end{aligned}
$$

Inserting the derivatives gives

$$
\begin{aligned}
\frac{-n}{\eta(n+1)}\cancel{(\ell_i^c)^2}c_i^{\cancel{2}} &> -q_i\cancel{(\ell_i^c)^2 c_i}\\
(n+1)\eta q_i &> nc_i
\end{aligned}
$$

which is already sufficed by the previous condition since $c_i < c_i + \tau_e e_i$.

To support the formulas used for the numerical illustrations as shown in Equations (31) to (33) the equations are solved explicitly here:

$$q_i c_i = \frac{n+1}{2n\ell_i^c}(1+\tau_c) \tag{A.6}$$

$$\frac{p_0 - c_i - \tau_e e_i}{\eta(n+1)} c_i = \frac{n+1}{2n\ell_i^c}(1+\tau_c) \tag{A.7}$$

$$\left(p_0 - c_i - \frac{\ell_i^c}{\ell_i^e} c_i\right) c_i = \frac{\eta(n+1)^2(1+\tau_c)}{2n\ell_i^c} \tag{A.8}$$

where Equations (9) and (13) have been used to replace q_i and e_i respectively. Equation (A.8) is a quadratic equation which is solved by Equation (31), where the lower of the two solutions has been chosen since the goal is to minimise costs.

Equation (32) can be calculated by inserting Equation (31) into Equation (13) and solving for e_i. q_i on the other hand, is calculated by inserting both, the solutions for c_i and e_i into the output equation in (9).

A.2. First Stage

The calculations hereafter show how the calculations to underline the results found in the calculation of the first stage. Starting with the benchmark scenario, we move on to the carbon tax and then the investment tax / subsidy.

A.2.1. Social

The calculation of the statements in Proposition 2 has already been shown in the corresponding section. To assure that the point calculated is actually a maximum, we need to see that the Hessian of the social welfare function is negative definite. Computing this Hessian matrix gives:

$$\begin{aligned}
H_U &= \begin{pmatrix} p'(Q) & -\frac{\mathrm{d}c_i}{\mathrm{d}x_i^c} & -\alpha\frac{\mathrm{d}e_i}{\mathrm{d}x_i^e} \\ -\frac{\mathrm{d}c_i}{\mathrm{d}x_i^c} & -q_i\frac{\mathrm{d}^2 c_i}{\left(\mathrm{d}x_i^c\right)^2} & 0 \\ -\alpha\frac{\mathrm{d}e_i}{\mathrm{d}x_i^e} & 0 & -\alpha q_i\frac{\mathrm{d}^2 e_i}{\left(\mathrm{d}x_i^e\right)^2} \end{pmatrix} \\
&= -\begin{pmatrix} \eta & -\ell_i^c c_i & -\alpha\ell_i^e e_i \\ -\ell_i^c c_i & q_i\left(\ell_i^c\right)^2 c_i & 0 \\ -\alpha\ell_i^e e_i & 0 & \alpha q_i\left(\ell_i^e\right)^2 e \end{pmatrix}
\end{aligned}$$

Its determinant computes to

$$\begin{aligned}\det H_U &= -\eta q_i^2 \left(\ell_i^c \ell_i^e\right)^2 c_i \alpha e_i + q_i \left(\ell_i^c \ell_i^e\right)^2 c_i \alpha e_i \left(\alpha_i e_i + c_i\right) , \\ &= q_i \left(\ell_i^c \ell_i^e\right)^2 c_i \alpha e_i \left(c_i + \alpha e_i - \eta q_i\right) .\end{aligned}$$

Since $\det H = \lambda_1 \cdot \lambda_2 \cdot \lambda_3$ where λ are the the eigenvalues of the matrix, this can only be negative definite if $\det H_U < 0$, so

$$\begin{aligned}q_i \left(\ell_i^c \ell_i^e\right)^2 c_i \alpha e_i \left(c_i + \alpha e_i - \eta q_i\right) &< 0 \\ c_i + \alpha e_i &< \eta q_i \\ \eta q_i &> c_i + \alpha e_i\end{aligned}$$

which is similar to the second stage solution except the factor $n/(n+1)$ and τ_e instead of α. This is also only valid if $q \neq 0$. The diagonal entries of the Hessian matrix also need to be negative but this is given since $\eta > 0$, $q_i > 0$, $c_i > 0$ and $e_i > 0$.

As in the part before, solutions to the implicit equations given in Equations (20) and (21) are given by the quadratic equations

$$c_i = \frac{1}{2\ell_i^c \left(\frac{1}{\ell_i^e} + \frac{1}{\ell_i^c}\right)} \left[p_0 - \sqrt{p_0^2 - 4n \cdot \eta \left(\frac{1}{\ell_i^e} + \frac{1}{\ell_i^c}\right)}\right] \tag{A.9}$$

$$e_i = \frac{1}{2\alpha\ell_i^e \left(\frac{1}{\ell_i^e} + \frac{1}{\ell_i^c}\right)} \left[p_0 - \sqrt{p_0^2 - 4n \cdot \eta \left(\frac{1}{\ell_i^e} + \frac{1}{\ell_i^c}\right)}\right] \tag{A.10}$$

$$q_i = 2\left(\frac{1}{\ell_i^e} + \frac{1}{\ell_i^c}\right) \left[p_0 - \sqrt{p_0^2 - 4n \cdot \eta \left(\frac{1}{\ell_i^e} + \frac{1}{\ell_i^c}\right)}\right]^{-1} \tag{A.11}$$

A.2.2. Carbon Tax

To make the written text of Section 4.3.2 better understandable, we first compute the derivative of the welfare function with respect to the carbon tax

$$\frac{\mathrm{d}U}{\mathrm{d}\tau_e} = \sum_{i \in \mathcal{F}} \left[\left(p_0 - \frac{\eta}{2}Q - c_i - e_i\right)\frac{\mathrm{d}q_i}{\mathrm{d}\tau_e} - \left(\frac{\eta}{2}\frac{\mathrm{d}Q}{\mathrm{d}\tau_e} - \frac{\mathrm{d}c_i}{\mathrm{d}\tau_e} - \alpha\frac{\mathrm{d}e_i}{\mathrm{d}\tau_e}\right) q_i - \frac{\mathrm{d}x_i^c}{\mathrm{d}\tau_e} - \frac{\mathrm{d}x_i^e}{\mathrm{d}\tau_e}\right] . \tag{A.12}$$

Highlighted in the text is the fact that the derivative of q_i, c_i and x_i^c with respect to the carbon tax are zero. Therefore Equation (A.12) greatly simplifies

$$\frac{\mathrm{d}U}{\mathrm{d}\tau_e} = \sum_{i \in \mathcal{F}} \left[\alpha\frac{\mathrm{d}e_i}{\mathrm{d}\tau_e} q_i - \frac{\mathrm{d}x_i^e}{\mathrm{d}\tau_e}\right] . \tag{A.13}$$

Taking the information contained in the implicit definition of the emissions in Equation (15), dividing this equation by $\tau_e q_i$ gives an estimate of e_i in dependence of τ_e. The derivatives present in Equation (A.13) compute to

$$\frac{\mathrm{d}e_i}{\mathrm{d}\tau_e} = \frac{n+1}{2n\ell_i^e\tau_e^2}(1+\tau_c)$$
$$\frac{\mathrm{d}x_i^e}{\mathrm{d}\tau_e} = \frac{1}{\ell_i^e\tau_e}$$

These give the equation presented in Equation (27).

A.2.3. Investment Tax / Subsidy

The investment tax / subsidy is arduous to calculate. For the sake of readability the tax is abbreviated hereafter exchanging $\tau = (1+\tau_c)$. For the derivative it is also unimportant whether it is taken with respect to τ or τ_c so we formulate everything in dependence of τ and calculate τ_c in the end. The general approach to the calculation is similar to the calculation of the carbon tax with the difference that every variable shows a dependence on τ. With some regrouping we can align the social welfare function to favour the insertion of Equations (14) and (15)

$$\begin{aligned} U &= \sum_{i\in\mathcal{F}} \left[\left(p_0 - \frac{\eta}{2}\right) q_i - c_i q_i - \alpha e_i q_i - x_i^e - x_i^c\right] \\ &= \sum_{i\in\mathcal{F}} \left[\left(p_0 - \frac{\eta}{2}Q\right) q_i - \frac{n+1}{2n}\left(\frac{1}{\ell_i^c} + \frac{\alpha}{\tau_e\ell_i^e}\right)\tau - x_i^e - x_i^c\right] \end{aligned} \tag{A.14}$$

now reformulating the first term of Equation (A.14) with the help of Equation (9) to favour the inclusion of Equation (14) gives

$$\begin{aligned} \left(p_0 - \frac{\eta}{2}nq_i\right) q_i = p_0 q_i - \frac{\eta\cdot n}{2}q_i^2 &= p_0 q_i - \frac{\cancel{\eta}\cdot n}{2}\frac{p_0 - m_\ell\ell_i^c c_i}{\cancel{\eta}(n+1)}q_i \\ &= p_0 q_i \frac{n+2}{2(n+1)} + \frac{\cancel{n}\cdot m_\ell}{2\cancel{(n+1)}}\frac{\tau\cancel{(n+1)}}{2\cancel{n}} \\ &= p_0 q_i \frac{n+2}{2(n+1)} + \frac{m_\ell}{4}\tau \end{aligned}$$

which leaves the derivative of social welfare with respect to the investment tax / subsidy

$$\frac{\mathrm{d}U}{\mathrm{d}\tau} = \sum_{i\in\mathcal{F}} \left[p_0\frac{n+2}{2(n+1)}\frac{\mathrm{d}q_i}{\mathrm{d}\tau} + \frac{m_\ell}{4} - \frac{n+1}{2n}\left(\frac{1}{\ell_i^c} + \frac{\alpha}{\tau_e\ell_i^e}\right) - \frac{\mathrm{d}x_i^c}{\mathrm{d}\tau} - \frac{\mathrm{d}x_i^e}{\mathrm{d}\tau}\right]. \tag{A.15}$$

To compute the other derivatives we reduce their dependence on τ to dependencies on cost with the help of Equations (9), (10) and (13). Starting with the outputs, be it total or of a single firm, the dependency on e_i is *eradicated* in favour of

$$q_i = \frac{p_0 - m_\ell \ell_i^c c_i}{\eta(n+1)} \tag{A.16}$$

where $m_\ell = (1/\ell_i^c + 1/\ell_i^e)$. Further, the investments can be reformulated as follows

$$\begin{aligned} x_i^c + x_i^e &= \frac{1}{\ell^c} \ln\left(\frac{c^0}{c}\right) + \frac{1}{\ell^e} \ln\left(\frac{e^0 \tau_e \ell^e}{\ell^c c}\right) \\ &= -m_\ell \ln c + \frac{1}{\ell^c} \ln(c^0) + \frac{1}{\ell^e} \ln\left(\frac{e^0 \tau_e \ell^e}{\ell^c}\right) \end{aligned}$$

where it is only the last term that shows a dependence on τ when taking the derivative.

Revisiting Equation (A.12) and including our now gained knowledge, we get

$$\begin{aligned} \frac{\mathrm{d}U}{\mathrm{d}\tau} &= \sum_{i\in\mathcal{F}} \left[\left(\frac{p_0(n+2)}{2(n+1)} \frac{-m_\ell \ell_i^c}{\eta(n+1)} + \frac{m_\ell}{c_i} \right) \frac{\mathrm{d}c_i}{\mathrm{d}\tau} + \frac{m_\ell}{4} - \frac{n+1}{2n} \left(\frac{1}{\ell_i^c} + \frac{\alpha}{\tau_e \ell_i^e} \right) \right] \\ &= \sum_{i\in\mathcal{F}} \left[m_\ell \left(\frac{1}{c_i} - \frac{p_0 \ell_i^c (n+2)}{2\eta(n+1)^2} \right) \frac{\mathrm{d}c_i}{\mathrm{d}\tau} + \frac{m_\ell}{4} - \frac{n+1}{2n} \left(\frac{1}{\ell_i^c} + \frac{\alpha}{\tau_e \ell_i^e} \right) \right] . \end{aligned} \tag{A.17}$$

To continue the calculation the derivative of c_i with respect to τ, we apply the implicit function theorem to Equation (14) and since its inverse is more useful to the forwarding of this calculation it is also diplayed here:

$$\begin{aligned} F(c_i, \tau) &= \frac{2n}{\eta(n+1)^2} \left[p_0 - m_\ell \ell_i^c c_i \right] \ell_i^c c_i - \tau = 0 \\ \Rightarrow \quad \frac{\mathrm{d}c_i}{\mathrm{d}\tau} &= -\frac{\frac{\mathrm{d}F}{\mathrm{d}\tau}}{\frac{\mathrm{d}F}{\mathrm{d}c_i}} = \frac{1}{\frac{2n\ell_i^c}{\eta(n+1)^2}(p_0 - 2m_\ell \ell_i^c c_i)} = \frac{\eta(n+1)^2}{2n\ell_i^c (p_0 - 2m_\ell \ell_i^c c_i)} \end{aligned} \tag{A.18}$$

$$\Rightarrow \quad \left(\frac{\mathrm{d}c_i}{\mathrm{d}\tau}\right)^{-1} = \frac{2n\ell_i^c (p_0 - 2m_\ell \ell_i^c c_i)}{\eta(n+1)^2} = \frac{2n\ell_i^c}{(n+1)} \left(2q_i - \frac{p_0}{\eta(n+1)} \right) \tag{A.19}$$

Acknowledging that the derivative in Equation (A.17) should vanish, we can reformulate it with the help of the carbon tax found before as shown in Equation (25).

$$
\begin{aligned}
m_\ell \left(\frac{1}{c_i} - \frac{p_0 \ell_i^c (n+2)}{2\eta(n+1)^2} \right) \frac{\mathrm{d}c_i}{\mathrm{d}\tau} &= \frac{n+1}{2n\ell_i^c} - \frac{m_\ell}{4} + \frac{n+1}{2n}\frac{\alpha}{\ell_i^e}\frac{2n}{(n+1)\alpha\tau} \\
m_\ell \left(\frac{1}{c_i} - \frac{p_0 \ell_i^c (n+2)}{2\eta(n+1)^2} \right) &= \left(\frac{n+1}{2n\ell_i^c} - \frac{m_\ell}{4} + \frac{1}{\ell_i^e \tau} \right) \left(\frac{\mathrm{d}c}{\mathrm{d}\overline{\tau}_c} \right)^{-1} \\
m_\ell \left(\frac{1}{c_i} - \frac{p_0 \ell_i^c (n+2)}{2\eta(n+1)^2} \right) &= \left(\frac{n+1}{2n\ell_i^c} - \frac{m_\ell}{4} + \frac{1}{\ell_i^e \tau} \right) \frac{2n\ell_i^c}{n+1} \left(2q_i - \frac{p_0}{\eta(n+1)} \right) \\
m_\ell - \frac{(n+2)\cdot m_\ell}{2(n+1)} \frac{p_0 \ell_i^c c_i}{\eta(n+1)} &= \left(\frac{1}{\ell_i^c} - \frac{n\cdot m_\ell}{2(n+1)} + \frac{2n}{(n+1)\ell_i^e \tau} \right) \left(2\ell_i^c c_i q_i - \frac{p_0 \ell_i^c c_i}{\eta(n+1)} \right)
\end{aligned}
$$

Grouping this into terms multiplied by $p_0 \ell_i^c c_i/(\eta(n+1))$, terms multiplied by $2\ell_i^c c_i q_i$ and the rest gives us more strucure:

$$
\begin{aligned}
T_1 &= \left(\frac{1}{\ell_i^c} - \frac{n\cdot m_\ell}{2(n+1)} + \frac{2n}{(n+1)\ell_i^e \tau} - \frac{(n+2)\cdot m_\ell}{2(n+1)} \right) \frac{p_0 \ell_i^c c_i}{\eta(n+1)} \\
T_2 &= m_\ell \\
T_3 &= \left(\frac{1}{\ell_i^c} - \frac{n\cdot m_\ell}{2(n+1)} + \frac{2n}{\ell_i^e \tau (n+1)} \right) 2\ell_i^c c_i q_i \\
\text{where} \quad & T_1 + T_2 = T_3
\end{aligned}
$$

Simplifying these individually:

$$
\begin{aligned}
T_1 &= \left(\frac{1}{\ell_i^c} - m_\ell + \frac{2n}{(n+1)\ell_i^e \tau} \right) \frac{p_0 \ell_i^c c_i}{\eta(n+1)} \\
&= \frac{1}{\ell_i^e} \left(\frac{2n}{(n+1)\tau} - 1 \right) \frac{p_0 \ell_i^c c_i}{\eta(n+1)} \\
T_3 &= \left(\frac{1}{\ell_i^c} - \frac{n\cdot m_\ell}{2(n+1)} + \frac{2n}{\ell_i^e \tau (n+1)} \right) 2\frac{(n+1)}{2n}\tau \\
&= \frac{n+1}{n}\frac{1}{\ell_i^c}\tau - \frac{m_\ell}{2}\tau + 2\frac{1}{\ell_i^e}
\end{aligned}
$$

Recombining the three terms gives

$$
\begin{aligned}
\frac{1}{\ell_i^e}\left(\frac{2n}{(n+1)\tau}-1\right)\frac{p_0\ell_i^c c_i}{\eta(n+1)}+m_\ell &= \left(\frac{n+1}{n}\frac{1}{\ell_i^c}-\frac{m_\ell}{2}\right)\tau+2\frac{1}{\ell_i^e} \\
\frac{1}{\ell_i^e}\left(2n-(n+1)\tau\right)\frac{p_0\ell_i^c c_i}{\eta(n+1)^2\tau} &= \frac{1}{2n}\left(\frac{2(n+1)-n}{\ell_i^c}-\frac{n}{\ell_i^e}\right)\tau+\frac{1}{\ell_i^e}-\frac{1}{\ell_i^c} \\
\left(2n-(n+1)\tau\right)\frac{p_0\left(\ell_i^c\right)^2 c_i}{\eta(n+1)^2\tau} &= \frac{1}{2n}\left((n+2)\ell_i^e\tau-n\ell_i^c\tau+2n\ell_i^c-2n\ell_i^e\right) \\
c_i &= \eta(n+1)^2\frac{(n+2)\ell_i^e\tau-n\ell_i^c\tau+2n\ell_i^c-2n\ell_i^e}{2np_0\left(\ell_i^c\right)^2\left(2n-(n+1)\tau\right)}\tau \\
c_i &= \eta(n+1)^2\frac{(2n-(n+1)\tau)(\ell_i^c-\ell_i^e)+(\ell_i^e+\ell_i^c)\tau}{2np_0\left(\ell_i^c\right)^2\left(2n-(n+1)\tau\right)}\tau \\
c_i &= \frac{\eta(n+1)^2}{2np_0\left(\ell_i^c\right)^2}\left(\frac{\ell_i^c+\ell_i^e}{2n-(n+1)\tau}\tau+(\ell_i^c-\ell_i^e)\right)\tau
\end{aligned}
\tag{A.20}
$$

To simplify this further we need to insert the solution for the costs in the second stages as given in Equation (31). Inserting the solution for c into Equation (A.20) computes to:

$$
\begin{aligned}
\frac{1}{\cancel{2\ell_i^c}m_\ell}\left(p_0-\sqrt{p_0^2-\frac{2m_\ell\eta(n+1)^2}{n}\tau}\right) &= \frac{\eta(n+1)^2}{\cancel{2}np_0\left(\ell_i^c\right)^{\cancel{2}}}\left(\frac{\ell_i^c+\ell_i^e}{2n-(n+1)\tau}\tau+(\ell_i^c-\ell_i^e)\right)\tau \\
\sqrt{p_0^2-\frac{2m_\ell\eta(n+1)^2}{n}\tau} &= p_0-\frac{\eta(n+1)^2m_\ell}{np_0\ell_i^c}\underbrace{\left(\frac{\ell_i^c+\ell_i^e}{2n-(n+1)\tau}\tau+(\ell_i^c-\ell_i^e)\right)}_{=T}\tau \\
\cancel{p_0^2}-\frac{2m_\ell\eta(n+1)^2}{n}\cancel{\tau} &= \cancel{p_0^2}-2p_0\frac{\eta(n+1)^2m_\ell}{np_0\ell_i^c}T\cancel{\tau}+\left(\frac{\eta(n+1)^2m_\ell}{np_0\ell_i^c}\right)^2T^2\tau^{\cancel{2}} \\
2\cancel{\frac{m_\ell\eta(n+1)^2}{n}} &= \cancel{2p_0}\frac{1}{\ell_i^c\cancel{p_0}}\cancel{\frac{\eta(n+1)^2m_\ell}{n}}T-\frac{\eta^{\cancel{2}}(n+1)^{\cancelto{2}{4}}m_\ell^{\cancel{2}}}{n^{\cancel{2}}p_0^2\left(\ell_i^c\right)^2}T^2\tau \\
2 &= \frac{2T}{\ell_i^c}-\frac{\eta(n+1)^2m_\ell}{np_0^2\left(\ell_i^c\right)^2}T^2\tau
\end{aligned}
$$

Reinserting the definition of T and solving for coefficients of τ gives

$$
\frac{\eta(n+1)^2}{2np_0^2}\frac{m_\ell}{\ell_i^c}T^2\tau = T-\ell_i^c
$$

$$
\frac{\eta(n+1)^2}{2np_0^2}\frac{m_\ell}{\ell_i^c}\left(\frac{\ell_i^c+\ell_i^e}{2n-(n+1)\tau}+\ell_i^c-\ell_i^e\right)^2\tau = \frac{\ell_i^c+\ell_i^e}{2n-(n+1)\tau}\tau-\ell_i^e \tag{A.21}
$$

Now reformulating the term $2n-(n+1)\tau=-n(\tau-2)-\tau$ and then multiplying by $(n(\tau-2)+\tau)^2$ gives

$$-\frac{\eta(n+1)^2}{2np_0^2}\frac{m_\ell}{\ell_i^c}\left(\ell_i^c+\ell_i^e-(\ell_i^c-\ell_i^e)(n(\tau-2)+\tau)\right)^2\tau=(\ell_i^c+\ell_i^e)(n(\tau-2)+\tau)+\ell_i^e(n(\tau-2)+\tau)^2 \tag{A.22}$$

Calculations from here on continue in the main part.

Declaration of consent

on the basis of Article 30 of the RSL Phil.-nat. 18

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