Intertemporal Allocation of Water Resources

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Abstract

Impacts of the on-going drought in California have created international and domestic concern regarding California's future freshwater resources. The current drought has affected statewide agriculture and environmental services and led to discussion in some municipalities concerning the use of desalination as a source for future freshwater. This thesis will highlight the complex, multi-dimensional interaction of California's intertied human and natural freshwater distribution and storage systems as well as discuss the influence of past droughts on the evolution of state water resources as well as discuss the potential of desalination. This thesis presents a social planner maximization problem, to analyze the benefit from which desalination could be derived. The model uses a decreasing freshwater influx parameter to replicate dry year influxes as well as consumption values within a range of the average urban municipal Californian water user. The model suggests that over time an increasing proportion of desalinated water will increase as the initial stock and resupply of the stock will be unable to meet demand due to freshwater extraction. Net benefit increase over time suggests that benefit from desalination would be produced over time. Desalination may be a supply-side option for regions that experience a complete loss of water resources. Regions that maintain annual precipitation and have an active agricultural industry may attempt to import embedded water for water intensive staple crops through virtual water trade.

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1. Introduction

Freshwater resources provide for a range of services across many intersecting and competing industries with usage and withdrawal varying between beneficiaries (Brauman et. al 2007). Global annual freshwater consumption is approximately 7450 cubic kilometers (km^3) of freshwater (Chapagain and Hoekstra 2004) with global demand to increase by 55% between 2000 and 2050 (OECD 2012). Drought and water scarcity highlight the devastating impact of increased global water demand. Global drought and water scarcity represent an international issue. This is compounded by an increase in global water demand due to population growth and development, supplemented by the uncertainty of climate change (Vörösmarty et al 2000**)**. Currently, climate projections estimate a reduction in renewable surface and groundwater resources in regions already vulnerable to drought (Jiménez et al. 2014).

One region already seeing the impacts of drought is California; the state is also likely to face climate-related water scarcity. California received international concern over the State's domestic freshwater resources. Highlighted by the usage of apocalyptic imagery of the near empty Shasta and Folsom water reservoirs, the effects of the drought beg the question: will California have enough freshwater to supply future demands? Water allocation has been a politically divisive issue throughout the history and geography of California and economic instruments have been created to attempt to allocate scare water resources as well as ensure environmental quality. Due to the States' temporal and spatial differences, drought management is complex and requires an integrated form of water management to respond to the challenges of multi-year droughts. Where California has suffered a historic multi-year drought, its winter storms have now replenished the State's reservoirs above historic averages. For the time being, water management policies and a proactive state government have worked to reduce the usage of all users and, at the same time, freshwater from winter and spring storms supplied by El Nino have increased freshwater reservoirs and icepack considerably compared to previous drought years.

However, the future of California's freshwater resources are less than certain; increased consumption and water withdrawal from a growing population and the agricultural industry will place limits on the usage of freshwater resources. Central to this, California's annual precipitation is observed as highly variable, both temporally and spatially in scale with long-term climate projections showing a decrease in heavy precipitation and mean precipitation within most areas of California (Bell et al. 2004). Given this, what options do California's water planners have to safeguard from future climate variability and water scarcity?

One strategy would be to manage the freshwater supply through implementing water policy on the state level. Adaptive and collaborative policy can address finite levels of uncertainty; historically State water policy has enabled an efficient use of water in drought conditions but water authorities and various sectors such as agriculture, municipal supplies and industry have consistently competed against one another for an increased share of water resources. Yet, policy can only enable an efficient usage of a decreasing resource and not produce an additional supply.

However theoretical new sources of exhaustible resources could be considered. A backstop technology, defined as "a new technology producing a close substitute to an exhaustible resource by using relatively abundant production inputs and rendering the reserves of the exhaustible resource obsolete when the average cost of production of the close substitute falls below the spot price of the exhaustible resource" (Khanna 2003). In the case of freshwater, backstops that can be considered would include desalination plants. Desalination plants are presently used in water scarce industrialized regions.

This thesis presents a cost benefit analysis using a renewable resource model to investigate how to maximize consumption of freshwater and inclusion of a backstop produced water supply, i.e., desalinated water, to a society and a consumer over infinite time periods.

This thesis is structured as follows:

- Chapters Two provides a summary of California's climate and the development of state water infrastructure. Then follows a description of desalination and the impact of drought on the state and histories of drought. Finally, the chapter will outline future climate impacts.
- Chapter Three deconstructs a renewable freshwater allocation model detailed and the accompanying outcomes.
- Chapter Four details freshwater supplies and demands in California while providing a description of water rights, and the water markets related institutions.
- Chapter Five discusses related models and their conclusions, whilst juxtaposing them against the model presented in the thesis with highlights the potential for desalination and virtual water in a water constrained future.
- Chapter Six is a summarizes the previous chapters and invites conclusions from the model.

2. Water Scarcity in California

To understand freshwater resources in California, this thesis will appreciate the complex, multi-dimensional interaction of intertied human and natural freshwater distribution and storage systems. This chapter will summarize the historical development of California's water systems, including desalination and its place within California's water allocation. In providing a summary, a brief history of droughts of the ongoing drought will be outlined and will ultimately contemplate on California's projected future climate impacts.

Central to this summary, it should be noted that drought and floods belong to two-sides of the same coin of hydrologic event extremes that exist in California. Flood events influence the historical development of state water systems, yet this thesis will focus solely on the impacts of drought as abundant sources juxtapose both topics in expanded depth.

2.1 California's Climate:

To discuss drought, this section will firstly reference the Western Regional Climate Centers 2001, *California's Climate*.

California's climate variability is directly influenced by its varied topography and general circulation patterns. It is generally consistent with a Mediterranean climate with rainy winters and dry summers.

Coastal ranges extend southerly from the State's northern border until meeting the Los Angeles basin. The northern coastal range merges with the Cascade mountain range with its southern counterpart merging with the traverse range, extending into the Mojave Desert. The Cascade mountain ranges then merge and extend into the Sierra Nevadas which form the eastern state boundary. The Cascade and Sierra Nevada ranges form the beginnings of the natural freshwater drainage system which extends throughout the Central valley and empties into the San Francisco Bay and eventually the Pacific Ocean. Mountains source much of the water supply as either rain or snow, with runoff supply reservoirs and snowpack; this delivers water supplies during the spring growing season. More than 70% of the streamflow is generated from the northern mountain ranges, whereas 80% of the water demand comes from the southern latitudes. Streamflow and total runoff is however dependent on precipitation and winter storms.

Precipitation is variable due to the conveyance or deflection influenced by a semi-permanent high pressure system. The Pacific high, in the north Pacific Ocean regulates the moisture and intensity of winter storms. Much of the annual precipitation occurs in the winter and is dependent on a small number of winter storms. Precipitation occurs as rainfall in lower elevations with snow deposited at higher elevations. Summer is generally dry over the entire state due to the northward migration of the Pacific high. Temperatures are usually moderate throughout the year but can vary due to microclimate and latitudes. Cold and hot extremes exist and depend on the local topography. Most years, dry summers persist with periods of little or no precipitation every year and require water resources to be stored and consumed for the drier periods of the year. The lack of winter storms can cause persistent interdecadal drought conditions and requires a spatially complex water conveyance system to meet state water demands. This disparity of water supply and water distribution has led to the development of California's state water conveyance and distribution systems.

2.2 Development of California's water system:

Development of the State of California's water distribution and storage systems began with initial investigations in the 1870s. Further planning occurred in the 1920s and 1950s, then implemented in the 1930s and 1980s[1]. The investigations in the 1870's were comprehensive and made recommendations for irrigation and flood control with improvements focused on the Central Valley for benefit of the state. Development of a statewide water project was proposed in 1919 with the goal of the project to transport water from the Sacramento River system, first to the San Joaquin Valley and then into Southern California. The Central Valley Project first began as a State project in 1933 but was taken over by the United States Federal government due to lack of funding in 1935. Following the post Second World War economic boom, statewide water demand increased due to increased agricultural and urban demand. The first California Water Plan, a comprehensive study of the state's hydrologic system was completed in 1957 and the development of a North-South water project was balloted and passed in 1960. Construction on the California Aqueduct began in 1963 with the first deliveries of water to Southern California made in 1973. The construction of the State Water project (SWP) consisted of 33 storage facilities, more than 1,100 kilometers (km) of canals and pipelines, 8 power plants producing 5100 Gigawatt hours (GWh) of electricity annually, with the SWP presently delivering drinking water to over 25 million people and 3000 km^3 of irrigated farmland (CDWR, 2011). Since the completion of the SWP, State water management has halted

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[1] A comprehensive history of California's water development and the State Water Project can be found here: http://www.water.ca.gov/swp/history.cfm as well as a general timeline here: http://www.watereducation.org/aquapedia/california-water-timeline, and the book Cadillac Desert by Marc Reisner who details water infrastructure and development across the Western U.S.

expansion of storage infrastructure and focused on policy measures, such as integrated water management (IWM) to increase resilience, management and water usage efficiency statewide.

State development around water policy has been shaped by responses to extreme events that aimed to ameliorate to both drought and extreme flooding. Traditional infrastructure development strategies have been more concerned on developing and providing storage and distribution infrastructure although periods of severe drought attract further investigations of unconventional sources of freshwater.

2.3 Backstop Technology: Desalination Plants

One source of freshwater that is resistant to both short term and long term climate variability comes from desalination plants. Desalination is the process by which saline water is converted into freshwater. Desalination, while it has its merits, has its disadvantages.

The benefits of seawater desalination interest water authorities and city managers, especially in areas reliant on variable precipitation. Firstly, desalinated freshwater can be reliably produced and are sympathetic to water budget considerations. Secondly, freshwater can be sourced throughout a drought, as freshwater produced from a desalination plant does not rely on the natural water cycle. Thirdly, local government or local water authorities can retain control of production rather than relying on interstate and Federal allocations. Freshwater produced at a desalination plant may not necessarily meet complete demand but will meet at least meet a fraction of municipal water budgets. Finally, the water quality of desalinated water can be controlled. Desalination plants process and treat all incoming water and can remove pollutants and contaminants that occur in streams and reservoirs thus producing a supply of potable drinking water.

There are hindrances that desalination plants will incur on municipalities and the environment if not sufficiently planned for. Firstly, the siting of a desalination plant requires careful consideration and input from many stakeholders. One cannot simply place a desalination plant and believe that it is optimal in location; desalination plants require large amounts of energy and have an impact on the local environment. Depending on the technology and the location, desalination plants impact local ocean water quality; discharge from the plant can lead to brackish conditions which leave dead zones in local underwater areas. Desalination plants are expensive and it is often difficult to compare the total cost depending on their location. For example, subsidies impact the capital requirement, therefore a plant in which the government freely provides land compared to a plant which requires expensive land to be purchased will impact the total cost of the plant. The costs of desalinated water will always be more expensive than freshwater delivered from the natural hydrologic system or pre-built conveyance systems. Energy is required for the processing of saltwater, and energy costs are variable. The price of energy will account for one-third to more than one-half the cost of produced water (Chaudhry 2003).

In many cases, cost-effective conservation and efficiency improvements can still increase water supply instead of desalination plants.

Currently ten plants operate in California with another nine proposed facilities[2], with the majority (57%) of installed capacity designed for municipal consumption (Cooley 2006). Desalinated water would satisfy 6% of California's year 2000 urban water demand if all proposed facilities were operational (Cooley 2006). Current plants account for less than 0.5% of urban water demand and are used to supply high-quality water for industrial locations (Cooley 2006). The most recently constructed desalination plant, the Carlsbad desalination plant completed in December 2015, was proposed by the San Diego Water Authority following decades of discussions about future water supply. Initial investigations were in response to the 1987-1992 drought, in which the county water supply was decreased by 33% from State water suppliers. Proposed desalination plants look to fulfil the role of a reliable freshwater source in municipalities hesitant to rely on State and Federal allocations, while at the same time maintaining a drought resistant source of freshwater.

2.4 Historic Droughts in California

This remaining section will discuss the impact of droughts on California's economy and non-market damages, whilst highlight past severe droughts with a description of the 2012 – 2017 ongoing drought (U.S. National Climate Data Center 2014).

2.5 Impacts of Drought in California:

California's climate is highly variable both spatially and temporally; average precipitation is dependent on a relatively small number of winter storms which can determine whether the year

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[2] Locations of current seawater desalination plants can be found here:
https://www.google.com/maps/d/u/0/viewer?mid=12fw0sl9pmbCN5FCbPeSEK0aIv3Q&ll=34.93836900983308%2C-119.6184915&z=8
As well as locations of proposed seawater desalination plants that can be found here:
https://www.google.com/maps/d/u/0/viewer?mid=14XCv4IBk7WMuNj7ICaxHGiKc6D8&ll=34.963876265526565%2C-118.36754266406251&z=8

will be dry or wet (CDWR 2016). Historically droughts have affected California on an interdecadal frequency but due to the subjective nature of identifying a drought, periods affected by drought are a matter of interpretation even in retrospect (Paulson 1989). The California Department of Water Resources (CDWR) defines a drought as a "water shortage for a particular user in a particular location.[3]" Similarly, the definition of a drought may be defined by the level of consumer, from urban water retailers that must predict and supply municipalities to the farmer who must plan for future irrigation and budget water costs.

Droughts have direct market impacts. Direct market impacts include but are not limited to: increased water surcharges, increased crop fallowing, direct loss of crop revenue and loss of employment.

Increased water surcharges affect consumers on all levels. In 2014, the peak of the recent drought, one counties' water surcharge increased by 785%, for per acre-foot (1233 m^3) of water sold, from $140 to $1,100 (Vekshin 2014). Some water districts experienced peak prices of up to $2,000 per acre-foot of water. Due to increased water surcharges, crop fallowing occurs in areas most impacted by decreased surface water; due to reliance on cheaper water, farmers will leave land fallow or even sell their water allotment. One sector particularly impacted are dairies and livestock, which comprise 15% of statewide agricultural revenue and rely on 6000 km^2 of irrigated hay and silage (Medeillin-Azuara et al. 2016).

Direct loss of crop revenue occurs from two causes, drought-related fallowing and crop pattern adjustments (Medeillin-Azuara et al. 2016). When direct impacts occur in areas impacted by a direct loss of crop revenue, goods and services are directly impacted which can lead to a loss of employment.

Other non-market damages include: groundwater overdraft and its associated issues, as well as decreased air quality including increased dust and visibility issues (Medeillin-Azuara et al. 2016).

Groundwater overdraft occurs when groundwater extraction exceeds the recharge rate into an aquifer. Groundwater is important as it accounts for nearly 40% of the water used in irrigation and depending on the region accounts for more than 90% of irrigation withdrawals in ten counties (USGS 2014). Groundwater overdraft leads to various subsurface problems such as: the accumulation of salt and other pollutants in groundwater, saltwater intrusion, land

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[3] There are three definitions, a meteorological drought – measured by the lack of precipitation, an agricultural drought, measured by the lack of soil moisture, or a hydrologic drought measured by the reduction in streamflow or groundwater levels. Taken from: U.S.G.S https://ca.water.usgs.gov/data/drought/

subsidence, damages to infrastructure, harm to groundwater dependent ecosystems, increased energy costs and economic losses from an unreliable water source (Cooley et al. 2015).

Beyond market and non-market impacts, droughts have influenced statewide development of institutions and infrastructure.

2.6 History of Droughts: Business as usual?

Historic droughts as determined by the CDWR have occurred from 1928-1934, 1976-1977, 1987-1992 and 2007-2009. Several of these droughts have caused the State to respond with additional measures:

- Drought in the 1920's and 1930's - The severity of this drought led to the creation of the Central Valley Project.
- Drought in the 1950's – The drought influenced investigations and legislation that led to the creation of the State Water Project.
- Drought in the 1970's – The drought led to the first urban conservation efforts. 1977 was the driest year on record.
- Drought in the 1980's - The drought resulted in the State's Drought Emergency Water Bank. The Emergency Water Bank provides a specific reservoir capacity to fulfil water needs in severe drought.
- Drought in the 1980's to the 1990's – The drought influenced the City and County of Santa Barbara as well as the County of San Diego to pursue and develop their own desalination plants.
- Drought in 2007-2009 – Persistent dry conditions result in one of the costliest wildfires in state history across a wide range of Southern California. The State of California declared the first state of emergency due to drought. Passage of the Water Conservation Act of 2009 required a 20% reduction in urban per capita water use.

Impacts of the on-going drought have affected agriculture and environmental services state wide. Beginning in 2011, the drought caused the lowest annual and 12-month precipitation, as well as the highest annual temperature and the most extreme drought indicators in the State's history (Diffenbaugh 2015). The drought has resulted in the driest three-year period in the State's history from 2012-2014 (CDWR 2015). Almost every month between December 2011 and September 2015 exhibited multiple indicators of drought (Diffenbaugh 2015).

2014 was the most significant year of the drought for several reasons. It was the third driest year on record. It was the first year that the SWP would halt deliveries due to low snowpack and reservoir levels (Boxall 2014). The total estimated impact of drought in 2014 cost the agriculture sector $1.5 billion with an estimated 5% of irrigated cropland lay fallow (Kerlin 2014). This included the loss of 17,100 seasonal and part-time jobs; most of the damages were incurred by counties that relied on surface water allocations (Howitt et al. 2014). 2014 was also notable because legislation was created in response to groundwater overdraft. A groundwater sustainability program was created in response due to increased groundwater pumping and a decrease of ground water levels, some areas experienced up to 30 meters of groundwater decrease (CDWR 2015). California was the last state to enact a framework for groundwater management. The Sustainable Groundwater Management Act (SGMA) passed in 2014 gave authority to the CDWR to establish fees and evaluate, and implement a statewide sustainable groundwater management plan.

However, the drought has continued through 2016 but could possibly end in 2017. Winter storms from 2016 to 2017 have increased reservoirs past historical averages (CDEC 2017) and the SWP has increased statewide water allocation by 20% from previous years (CDWR 2017).

> *"Historically, California's significant multi-year droughts have been ended by an above-average water year where statewide precipitation was in the range of 150 percent of average. Because California's annual water budget is determined by only a small number of winter storms, having a significantly above average year translates to having a winter season with a few very large storms. On average, about half of California's average annual precipitation occurs from December through February, which coincides with the typical timing of the largest winter storms." (CDWR)*

But does the severity of the drought indicate a new norm or provide any insight into future climate in California?

2.7 Future Climate in California

Overall projections of future climate in California tend toward a warmer, drier climate. The projected rate of warming is accelerated in a distinctive way from the historical rates estimated from observed temperature records in California (Bonfils et al. 2008). Cayan et al. 2013 found that results from a sixteen CIMP3 ensemble models resulted in warming in both low and high emission scenarios. For low-emissions scenarios, the amount of warming ranges from 0.6° Celsius(C) to 1.7 ° C for the period 2021-2050, 0.6°C to 2.2°C for 2041-2071 and 1.1°C to 3.3°C for 2070-2099. High-emission scenarios project warming from 1.1 to 2.2°C for the period 2021-2050, 1.1°C to 3.3°C for 2041-2070 and 2.8°C to 5°C for 2070-2099.

Increased temperatures do have several impacts on California's water resources. Firstly, increased temperatures cause more precipitation to fall as rain instead of snow, with impacts of reducing overall snowpack by up to 70 to 90% (Dettinger et al. 2015). The snowpack of 2015 was the lowest on record for at least the last 500 years (Belmecheri et al. 2015). This is important, as the SWP relies on snowpack to supply water during the dry spring and summer months. Secondly, increased temperatures would lead to warmer years with more severe heat, increasing the amount of days over 37°C by up to 100 (CCCC 2016).

Depending on the period, global climate model (GCM) and scenario chosen, projections tend to agree in a decrease in future annual precipitation (from -37 mm/yr to -157 mm/yr) as well as snowpack at all elevations (from -26% to -89%) (Hayhoe et al. 2004). Long-term climate projections show a decrease in heavy precipitation and mean precipitation within most areas of California (Bell et al. 2004). However, models project changes in precipitation equal to natural variability (Cayan et al 2013) and changes in future precipitation are small when compared to inter annual and intermodel variability (Pierce et al. 2013). Models show that the Mediterranean seasonal precipitation patterns will continue with most precipitation occurring in the winter (Cayan et al. 2013). Changes in precipitation are important however, as consumers are dependent on historic levels of precipitation and current water resources are fully utilized.

Diffenbaugh et al. (2015) modeled California's future climate and the impact of warming on the likelihood of drought. Furthermore, they found that anthropogenic warming is increasing the likelihood of drought, due to a co-occurrence of warm-dry conditions. Another conclusion being that precipitation deficits in California were more than twice as likely to yield drought years if they occurred when conditions were warm and that the probability of precipitation deficits to produce a drought have also increased. Diffenbaugh found that the occurrence of drought years has been greater in the past two decades than in the preceding century.

3. Intertemporal Allocation of Water

The following chapter includes a renewable resource model, the description of its numerical calibration and outcomes from the model.

3.1 Renewable Resource Model

The cost benefit analysis that follows has several objectives: the social planner attempts to maximize the benefit to consumers, as a function of total water extracted and desalinated while minimizing the cost of extraction and production of backstop water.

3.2 Model Framework:

The model will require several assumptions:

1. Information is costless and readily available.
2. Stocks of water, x_t, resources are non-perishable.
3. Extraction of water, y_t, occurs at constant marginal costs, α.
4. In addition, water from desalination, z_t, can be provided at costs $\frac{\beta}{2} z_t^2$.
5. $(y_t + z_t)$ equals the total amount of water available to a society at period *t*.

The following variables will also apply to the social planner optimization: ε_t represents the freshwater influx at period *t*. Freshwater influx ε_t is also exogenously given as $\varepsilon_t = \varepsilon_0 \cdot {R_\varepsilon}^t$, with $0 < {R_\varepsilon}^t < 1$. The rate of influx will be between 0 and 1, and will result in decreasing freshwater influx as time progresses. We obtain individual water consumption, w_t, where w_t, is the sum of freshwater, y_t, and desalinated water, z_t.

$$w_t = y_t + z_t.$$

Benefits of water consumption are given by the benefit function, $B(w)$. Society is populated by *N*, identical individuals, each of which has a benefit of $B(w_t)$ from water consumption w_t.

$$B(w_t) = \Gamma w_t(\Phi - \tfrac{1}{2} w_t)$$

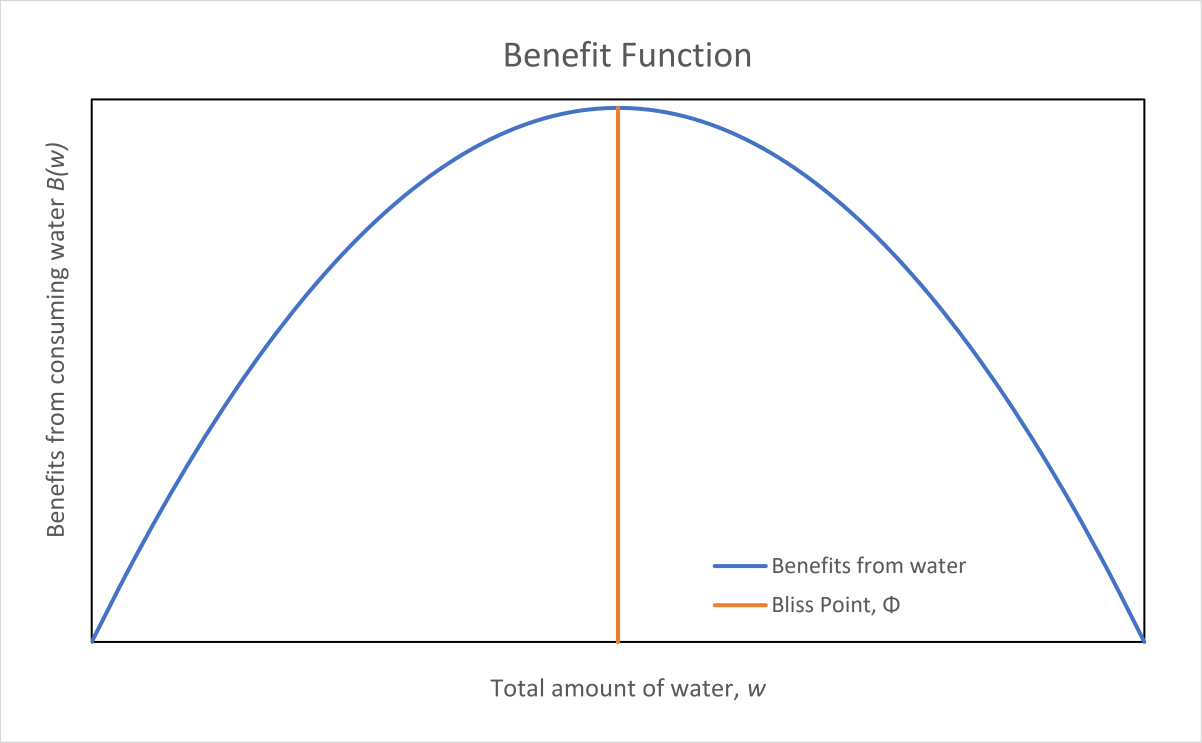

Figure 1. An example of the quadratic benefit function, B(w), the orange line, Φ, represents the bliss point and the blue line, B(w), represents the benefits from water. This graph does not use the same numbers from the model.

Fig. 1 illustrates how the benefit function behaves. Φ represents the bliss point, a point at which any increased or decreased consumption will yield less utility for an individual. Γ, is the benefit from water utility, this will be expressed in a dollar ($) per volume unit and is exogenously given.

Table 1 Exogenous and endogenous variables used in the renewable water model.

Table 1. Exogenous variables

t	A period or unit of time
N	Population or number of individuals, constant
β	Marginal cost of desalination
α	Marginal cost of extraction
ε_t	Freshwater influx at *t*
ε_0	Freshwater at *t=0*
Φ	Bliss point
Γ	Benefit from water utility

R_ε	Rate of Freshwater influx, $0 < R_\varepsilon < 1$
x_0	Initial stock of freshwater
δ^t	Discount factor, at *t*. $0 < \delta < 1$
	Endogenous Variables
w_t	Benefit in period *t*
y_t	Freshwater extraction at *t*
z_t	Desalinated water produced at *t*
x_t	Water stock at period *t*
z_0	Desalinated water produced at *t*=0
T	Period at which freshwater extraction will only ever equal freshwater influx

Table 1 displays exogenous and endogenous variables. The social planner wishes to maximize the discounted sum of benefits (net of costs) over an infinite time horizon. Using a discount factor, δ^t, which prioritizes benefit in earlier periods as compared to later periods, with $0 < \delta < 1$.

First the Lagrangian must be set up, we must first look at the objective function:

$$\underset{y_t, z_t}{\text{Max}} \sum_{t=0}^{\infty} [N \cdot B\left(\frac{y_t + z_t}{N}\right) - \alpha y_t - \frac{\beta}{2} z_t^2]\delta^t$$

$$\text{s.t } x_{t+1} = x_t - y_t + \varepsilon_t \text{ , } x_0 \text{ given and } \varepsilon_0 \text{ given}$$

At which point we come to the Lagrangian by constraining for x_{t+1}, thus:

$$L = \sum_{t=0}^{\infty} \delta^t \, [N \cdot B\left(\frac{y_t + z_t}{N}\right) - \alpha y_t - \frac{\beta}{2} z_t^2 + \delta \lambda_{t+1}(x_t - y_t + \varepsilon_t - x_{t+1})]$$

Then we will derive the first order conditions (FOCs) by taking the partial derivative w.r.t. $y_t, z_t,$ and x_t:

1. $\frac{\partial L}{\partial y_t} = \delta^t \left[N \cdot B'\left(\frac{y_t + z_t}{N}\right) \cdot \frac{1}{N} - \alpha - \delta \lambda_{t+1}\right] = 0 \rightarrow B'(w_t) = \alpha + \delta \lambda_{t+1}$

The first FOC yields the optimal amount of freshwater extraction, w.r.t. to the benefit derived from the consumption of freshwater, and the costs that are incurred from the extraction of freshwater. The right-hand side (RHS), $\alpha + \delta \lambda_{t+1}$, will yield positive as will the left-hand side(LHS) so that the benefit will increase w.r.t. w_t, however as w_t approaches and increases beyond Φ, benefit will decrease.

2. $\frac{\partial L}{\partial z_t} = \delta^t \left[N \cdot B'\left(\frac{y_t+z_t}{N}\right) \cdot \frac{1}{N} - \beta z_t \right] = 0 \ \rightarrow \ B'(w_t) = \beta z_t$

The second FOC yields the optimal amount of desalinated extraction w.r.t the benefit derived from the consumption of desalinated water, and the costs incurred from the extraction of desalination.

The LHS, $B'(w_t)$, the rate of change of the benefit w.r.t. change in w_t, will be negative if $w_t > \Phi$. At some point, there will no additional demand for w_t and can be hurtful. The RHS, is positive as both β and z_t, are positive.

3. $\frac{\partial L}{\partial x_t} = \delta^t[\delta \lambda_{t+1} - \lambda_t] = 0 \ \rightarrow \lambda_t = \ \delta \lambda_{t+1}$

The third FOC is used as a parameter for the Lagrangian and thus has no economic insight.

Following derivation of the FOCs[4], we will find the solutions for desalinated water, z_t and freshwater extract, y_t, which act as a function of y_t and z_t, respectively:

$$z_t = \frac{\Gamma(N\Phi - y_t)}{N\beta + \Gamma}$$

$$y_t = \ N\Phi - \frac{N\Phi\beta + \Gamma}{\Gamma} z_t$$

Which allows us to solve for desalinated water in the following period, z_{t+1}:

$$z_{t+1} = \frac{1}{\delta} z_t + \frac{\alpha}{\beta}(1 - \frac{1}{\delta})$$

This results in a system of two difference equations for desalinated water, z_t and the water stock, x_t, following:

1. $z_{t+1} = \ \frac{1}{\delta} z_t + \frac{\alpha}{\beta} \ (1 - \frac{1}{\delta})$
2. $x_{t+1} = x_t - N\Phi + \left(\frac{N\Phi\beta + \Gamma}{\Gamma}\right) z_t + \varepsilon_t$

And allows us to find the solution for desalinated water, z_t, as constrained by desalinated water in the initial period, z_0:

$$z_t = z_0 \delta^{-t} + \frac{\alpha}{\beta}(1 - \delta^{-t})$$

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[4] Proof is in Appendix A: Renewable resource model

We then solve the system of difference equations, which first leads to the general solution of desalinated water, z_t, as a function of desalinated water in the initial period, z_0:

$$z_t = z_0\delta^{-t} + \frac{\alpha}{\beta}(1-\delta^{-t})$$

Then by inserting, desalinated water, z_t into the water stock of the following period, x_{t+1}, we will arrive at the solution for the water stock, x_t as a function of the initial water stock, x_0 and the initial volume of desalinated water, z_0.

$$x_{t+1} = x_t - N\Phi + \left(\frac{N\Phi\beta + \Gamma}{\Gamma}\right)\left[\frac{\alpha}{\beta} + \left(z_0 - \frac{\alpha}{\beta}\right)\delta^{-t}\right] + \varepsilon_t$$

And as a reminder, $\varepsilon_t = \varepsilon_0 \cdot {R_\varepsilon}^t$. Thus, x_{t+1} as a function of x_t and z_0.

$$x_{t+1} = x_t + \left(N\frac{\beta}{\Gamma} + 1\right)\left(z_0 - \frac{\alpha}{\beta}\right)\delta^{-t} + \varepsilon_0 \cdot {R_\varepsilon}^t + \left(N\frac{\beta}{\Gamma} + 1\right) - N\Phi$$

By solving for a recurrence relation, we can solve for several steps which leads us to the solution for x_t, constrained by x_0.

$$x_t = x_0 + \frac{\delta}{1-\delta}\left(N\frac{\beta}{\Gamma} + 1\right)\left(z_0 - \frac{\alpha}{\beta}\right)(\delta^{-t} - 1) + \frac{\varepsilon_0}{R_\varepsilon - 1}\left({R_\varepsilon}^t - 1\right) + [\left(N\frac{\beta}{\Gamma} + 1\right)\frac{\alpha}{\beta} - N\Phi] \cdot t$$

With x_0, equal to:

$$x_0 = A + \frac{\delta}{1-\delta}\left(N\frac{\beta}{\Gamma} + 1\right)\left(z_0 - \frac{\alpha}{\beta}\right) + \frac{\varepsilon_0}{R_\varepsilon - 1}$$

And A equal to:

$$A = x_0 - \frac{\delta}{1-\delta}\left(N\frac{\beta}{\Gamma} + 1\right)\left(z_0 - \frac{\alpha}{\beta}\right) + \frac{\varepsilon_0}{R_\varepsilon - 1}$$

Recalling z_t as a function of z_0, $z_t = z_0\delta^{-t} + \frac{\alpha}{\beta}(1-\delta^{-t})$, we can insert z_t into the solution for y_t to find y_0 and z_0:

$$y_t = N\Phi - \frac{N\Phi\beta + \Gamma}{\Gamma} \cdot [z_0\delta^{-t} + \frac{\alpha}{\beta}(1-\delta^{-t})]$$

$$y_0 = N\Phi - \frac{N\Phi\beta + \Gamma}{\Gamma} \cdot z_0$$

$$z_0 = \frac{N\Phi - y_0}{N\Phi\frac{\beta}{\Gamma} + 1}$$

Plugging z_0 in to y_t to solve for y_t:

$$y_t = N\Phi - (N\Phi - y_0)\delta^{-t} - \frac{\alpha}{\beta}\frac{N\beta + \Gamma}{\Gamma}(1 - \delta^{-t})$$

Simplified to:

$$y_t = y_0\delta^{-t} + [N\left(\Phi - \frac{\alpha}{\beta}\right) - \frac{\alpha}{\beta}](1 - \delta^{-t})$$

We can then abbreviate by introducing *K*, all terms being constant:

$$K = N\left(\Phi - \frac{\alpha}{\Gamma}\right) - \frac{\alpha}{\beta}$$

Which allows to simplify the solutions for: x_t, y_t, z_t and express them in terms of y_0 and x_0.

$$x_t = x_0 - y_0\frac{\delta(\delta^{-t} - 1)}{1 - \delta} + \varepsilon_0\frac{\varepsilon_0}{R_\varepsilon - 1} + K[\frac{\delta(\delta^{-t} - 1)}{1 - \delta} - t]$$

$$y_t = y_0\delta^{-t} - K(\delta^{-t} - 1)$$

$$z_t = \frac{N\Phi - y_0}{N\frac{\beta}{\Gamma} + 1}\delta^{-t} + \frac{\alpha}{\beta}(1 - \delta^{-t})$$

Solving for y_0, we obtain an equation that is not analytically solvable, thus we must numerically determine, *T*, the point at which freshwater influx will equal extraction. There exists some time, *T*, for which holds, $y_T = \varepsilon_T, \forall\ t \geq T$.

$$y_T = y_0 \cdot \delta^{-T} - K(\delta^{-T} - 1) = \varepsilon_0 \cdot {R_\varepsilon}^T$$

$$y_0 = \varepsilon_0 \cdot {R_\varepsilon}^T \cdot \delta^T + K(1 - \delta^T)$$

$$y_t = \varepsilon_0 \cdot {R_\varepsilon}^T \cdot \delta^{T-t} + K(\delta^{-t} - \delta^{T-t}) - K(\delta^{-t} - 1)$$

$$y_t = \varepsilon_0 \cdot {R_\varepsilon}^T \cdot \delta^{T-t} + K(1 - \delta^{T-t})$$

After *T*, there will no longer be any available stock of water to drawn from and thus extraction must equal to the influx in each period.

To solve for the period *T*¸ we will first solve the LHS and the RHS using the following equation:

$$x_0 + \sum_{t=0}^{T-1} \varepsilon_t = \sum_{t=0}^{T-1} y_t$$

$$x_0 + \varepsilon_0 \sum_{t=0}^{T-1} \cdot {R_\varepsilon}^t = \sum_{t=0}^{T-1} [\varepsilon_0 \cdot {R_\varepsilon}^T \cdot \delta^{T-t} + K(1-\delta^{T-t})]$$

$$x_0 + \varepsilon_0 \frac{{R_\varepsilon}^T - 1}{R_\varepsilon - 1} = \varepsilon_0 \cdot {R_\varepsilon}^T \cdot \delta^T \frac{\delta}{1-\delta}(\delta^{-T} - 1) - K\delta^T \cdot \frac{\delta}{1-\delta}(\delta^{-T-1}) + KT$$

Which leads the following equation to solve the model:

$$x_0 + \varepsilon_0 \frac{{R_\varepsilon}^T - 1}{R_\varepsilon - 1} = \varepsilon_0 \cdot {R_\varepsilon}^T \frac{\delta}{1-\delta}(1 - \delta^T) - K \cdot \frac{\delta}{1-\delta}(1 - \delta^T) + KT$$

3.3 Numerical Calibration

The model has been calibrated to reflect an annual per capita outcome of an average Californian urban municipal water user. The population of the model is an individual, *N*, set to 1. Values found in the model represent averages from statewide municipal districts apart from the bliss point.

Per capita satiation has been set to the amount of water consumption of an American in a non-water constrained state, which uses approximately 100 gallons of water per day. Thus, the bliss point , *Φ*, has been set to 36,500 (WaterSense 2008).

The marginal cost of extraction, α, was set to 400. The average monthly water bill varies for most urban users from between \$30 to \$60[5], annually this is approximately \$360 to \$720 per year. The average price of water delivered by State Water Project's is \$147 per m^3 (CDWR, 2011) or \$.55/gal. Urban users pay more per gallon than their rural counterparts who purchase water from the State Water Project. The urban municipal water consumer in this model will pay approximately \$.33/gal.

Values for the cost of desalination vary from location and typically depend on initial contract agreed upon by the municipality (Cooley 2006). Municipalities tend to order water in the thousands of gallons (kgal) volume with the cost of \$4.60/kgal produced by the Charles Meyer Desalination plant located in Santa Barbara. The production cost is unrepresentative of the costs incurred by water customers, so the cost of desalination has been set to \$5/gal.

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[5] Due to the structure of water authorities, the range of monthly fees, this is a gross approximation based off estimates from Wells 2015.

The discount rate, δ, has been set to 5%. Typically, the discount rate is set depending on the objectives set by the period in question or by calculating net present value. The discount rate in this model will be set to 5% since it is impossible to predict by which period the stock will have been extracted. Models in the following chapter use varying time periods, from one decade to one century, to analyze water usage in California.

The benefit from water utility, Γ, is difficult to estimate as valuation of water heavily relies on the consumer's relationship with water (Savenije 2002). The market for water is heterogenous, even across California and services and sectors that rely on water value the consumption of water differently. Urban water users do not depend nor value water for their livelihood as compared to the agricultural industry and therefore have a higher willingness to pay. The benefit has been set to represent a gallon of water that an individual would pay for in a supermarket at $2, which is still much higher per gallon that consumers in the United States pay[6].

The initial stock of water, x_0, was set to five times the amount of per capita usage. This reflects a similar capacity of California's reservoirs to withstand five years of drought. Similarly, the initial influx of water, ε_0, is set to 1/3rd of x_0. This is parameterized as a function of a dryer year. With the rate of freshwater influx, R_ε, set to 95% to represent a slowly decreasing supply of influx during a dry year.

Table 2. Given model values

Exogenous Model Values	
N	1
β	$4/gal
α	$400/
ε_0	66,000/gal per year
Φ	36,500 gal/per year
Γ	$2/gal
R_ε	.95
x_0	200000 gal
δ^t	95%

Table 3. Values calculated from given values.

Calculated Model Outcomes:	
T	55.9763496
RHS	1445248.02
LHS	1445248.02
RHS-LHS	8.2282E-07
y_0	34380.5124/gal
z_0	605.56789/gal

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[6] The average price for a gallon of bottled water in 2014 in the United States was $1.20. Source: the International Bottled Water Association, http://www.bottledwater.org/economics/real-cost-of-bottled-water.

3.4 Model Outcomes

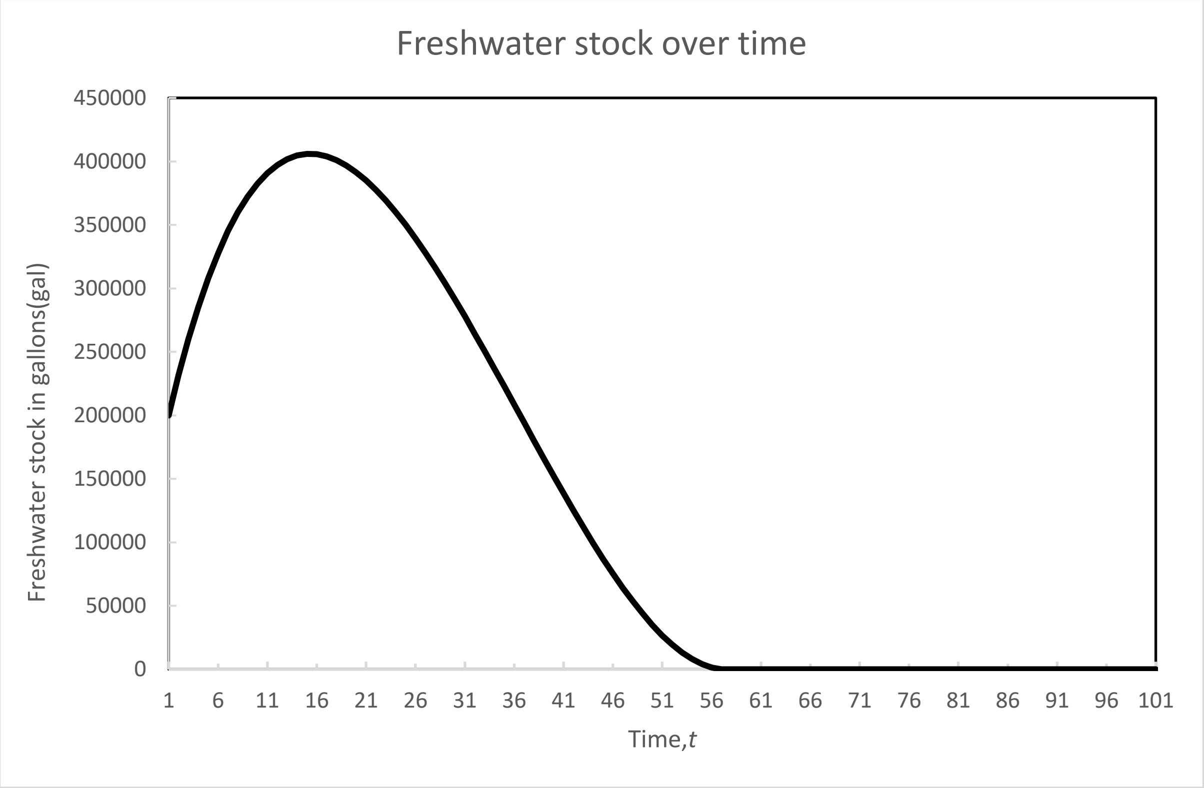


Figure 2. The evolution of the freshwater stock overtime. Depletion occurs due to freshwater extraction.

Fig 2. Displays the evolution of the freshwater stock, x_t. The freshwater stock is extracted at a rate which corresponds to *T*=55.9763496. The stock reaches its highest capacity at *t*=14 due to freshwater influx and small but increasing amounts of desalinated water but degrades until depletion at *T*. **Fig. 3** juxtaposes the decreasing freshwater influx with the change in freshwater extraction. Extraction of freshwater, y_t, slowly decreases overtime until the stock is completely exhausted and can only extract as much water as is delivered through freshwater influx. Once the freshwater stock is depleted and influx only represents a fraction of historic precipitation, then desalination results in the majority of net benefit **Fig. 4.**

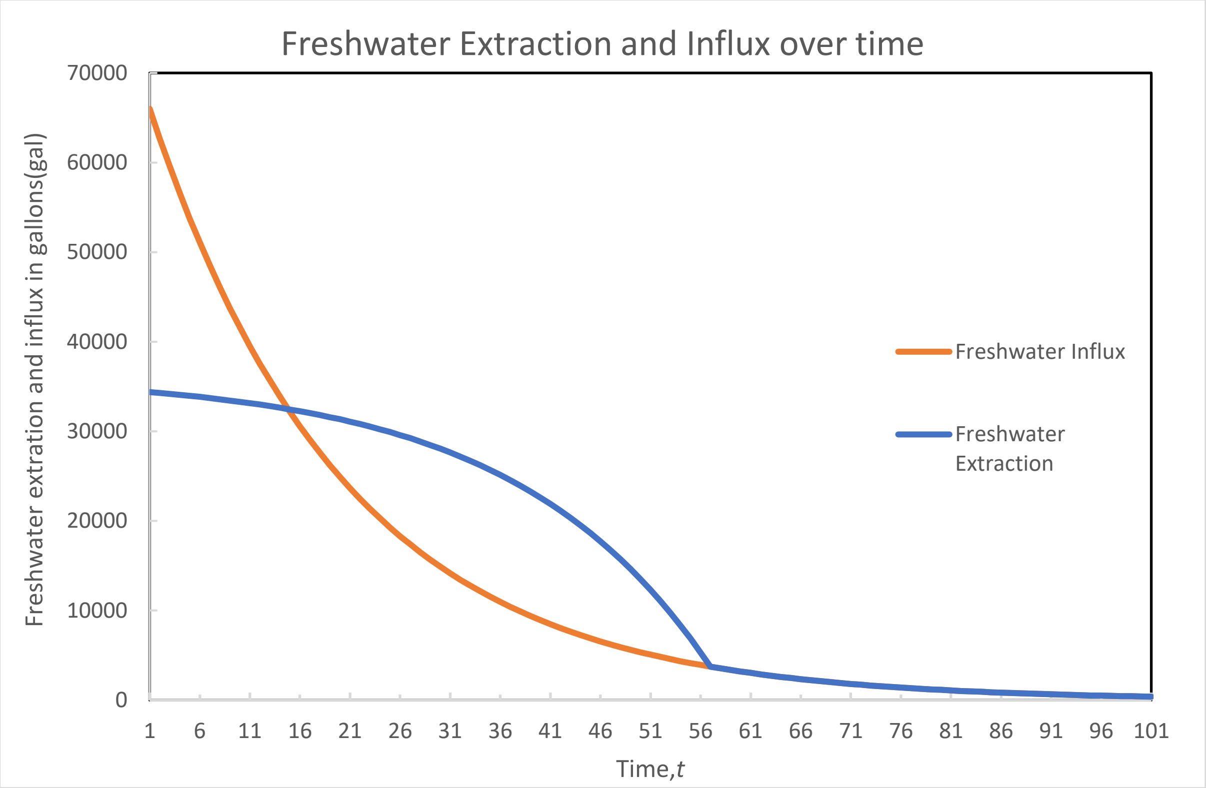


Figure 3. Displays freshwater extraction and influx overtime. Freshwater extraction can no longer sustain extractions from the water stock and can only rely on extraction based on that periods influx of freshwater.

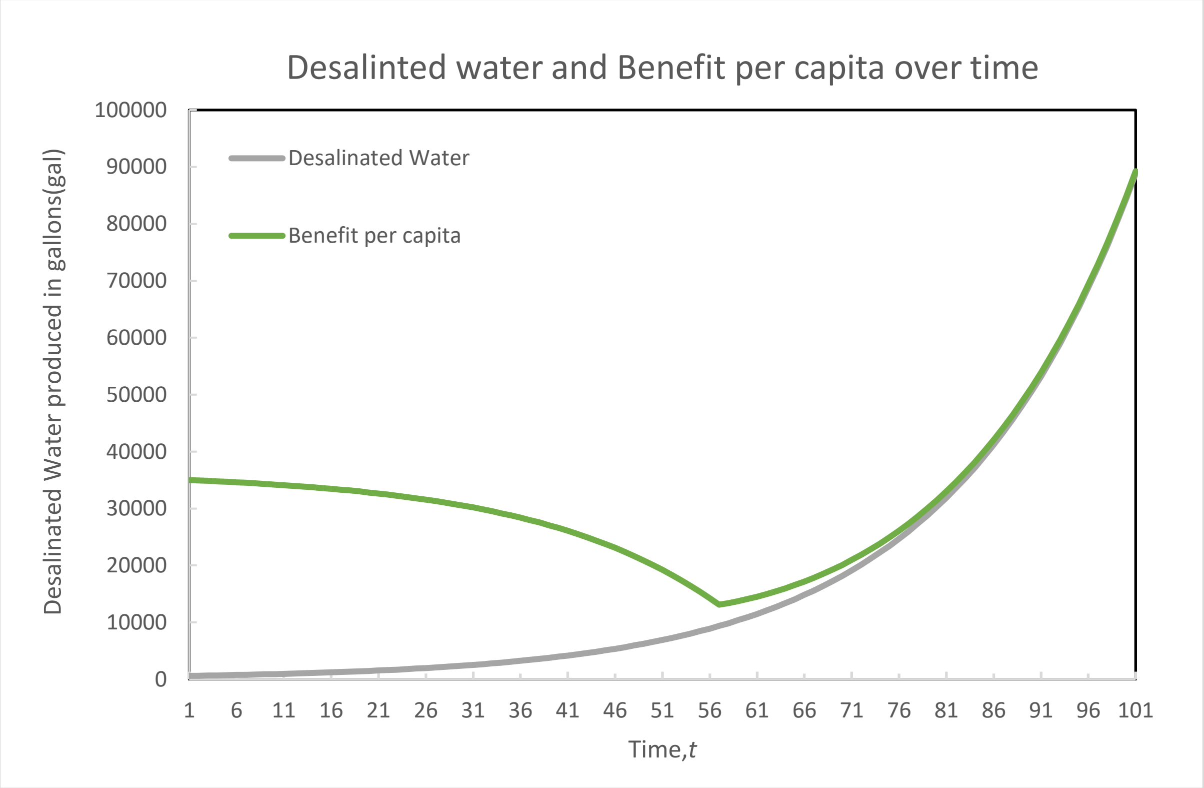


Figure 4. Displays the amount of desalinated water and net benefit per capita over time. After *T*, desalinated water becomes almost directly responsible for benefit per capita.

4. Distribution of Water in California

This chapter will first discuss the supply and demand of freshwater across the state, following will be a summary of California's water rights and then water market institutions.

The preceding model does not account for separate stocks of water. Real world conditions dictate that freshwater supply will be stored in different locations and originate from different sources. Although the model does include two sources of freshwater, either freshwater influx or through production of desalinated water this does not completely account for real world conditions.

4.1 Freshwater Supply

Annually an estimated 246 km^3 of precipitation is deposited across California, with most of the precipitation evaporated. The remaining 93 km^3 of unimpaired runoff flows into streams, groundwater basins, and surface water reservoirs. Most of the precipitation falls as rain and snow in the north. Major geographic disparities persist from north to south, with most runoff (2/3rd) originating from the wetter 1/5th of the state. The driest 1/3rd of the state produces 0.1% of total water availability (Hanak 2016a). The driest regions, include Southern California and the agricultural areas of the Tulare Basin and Imperial Valley, where water demands contrast with the gross annual runoff volume.

Statewide water demands are met by multiple water sources ranging from in-state precipitation, to imports from other states, groundwater extraction, as well as treated wastewater and desalination. **Fig. 5** shows the diversity of water sources for agricultural, environmental and urban water uses (Hanak 2016a). Surface waters make up 45% of water supply, with local and out of state streams making up 80% (before reuse and recycling) of the total supply. Groundwater represents approximately 18% of the total supply.

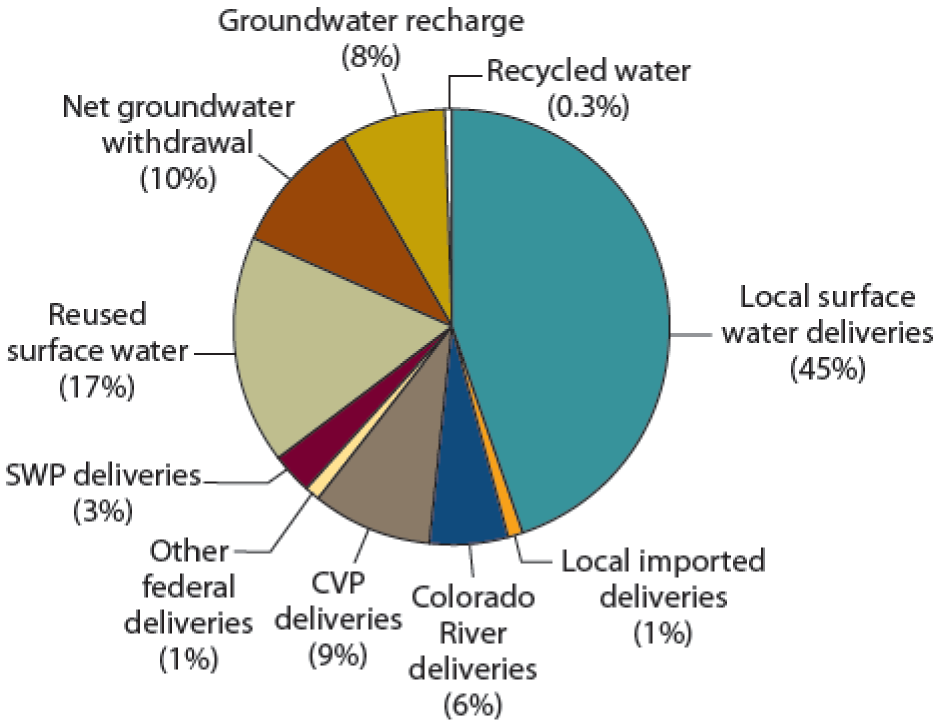


Figure 5. Displays the sources of gross water supplies during the 1998-2005 average. Source: California Department of Water Resources (2009), graph from Hanak 2016a.

4.2 Freshwater Demand

California's water system currently serves over 38 million and is expected to grow by 50 million by 2049 (Johnson 2016). Total present demand is difficult to estimate due to the lack of mandatory monitoring and reporting requirements as well as disputed estimation of demand. The three major categories of withdrawals can be divided into: agricultural (40%), urban (10%) and environmental demands (50%) (Hanak 2016a). The statewide totals are controversial, as farmers insist that the total supply for the environment is too high and that more water needs to be allocated to agricultural regions.

Statewide totals include a high percentage for environmental demands due to hydraulically isolated areas that use a majority of their water for the environmental purposes. Two such regions include the North coast and the North Lathontan. When statewide demand is corrected for these two regions, total net water demands from agriculture increase to 62%, as well as reduction in environment demand at 22% and urban approximately at 16%.

Even amongst sectors there is a geographic divide between users. Inland urban water users consume more than their coastal counterparts in part due to differences in climate. Inland dwellers live in areas with more average heating degree days and higher average temperatures. In dryer, desert areas water consumption is order of magnitudes higher. For example, Palm Springs has the highest per capita water consumption in the state driven by water usage in landscaping. Most consumption from households is due to landscaping rather (up to 50%) than any other purpose (Mercury 2014). The usage difference in farmers can result from the type of agriculture or type of crop planted.

4.3 California's Water rights

Water has been treated as a private good since the beginning of modern State history. Water rights have been protected since appropriations were recognized with the Supreme Court case *Irwin v. Phillips* in 1855. With water rights being held under the "first in time, first in right" legal system (Hanak 2012). This type of water right is known as an appropriative right, where water usage is based on the physical control and use of the water.

Water rights can be divided into two sources, surface water and ground water. Both surface water and groundwater follow a similar classification scheme, with seniority of appropriation dictating the priority of allocation. The three classes of surface water rights: riparian water rights, rights appropriated before the Water Commission Act of 1914 and rights appropriated after 1914. Riparian water rights are the most senior of the water rights and available to landowners directly adjacent to rivers, lakes and estuaries on that land. These rights are not available to cities or municipal water suppliers and generally also have the highest allocations. Rights appropriated before 1914, were claimed before the adoption of the water code in 1914. These rights are not tied to property rights but are tied to the historical usage of the initial appropriation. Rights appropriated after 1914 are subject to more direct oversight than their historical superiors. Rights claimed more recently will be curtailed or reduced completely during periods of water shortages. Rights can be bought and sold on the water market pending oversight from the local and state water board.

4.4 California’s Water Market Institutions

Water authorities

The State Water Resources Control Board, colloquially called “the water board”, has authority to issue rights, yearly allocations and curtailments although this authority is not definitive. Challenges to its authority by senior water right’s holders and upheld by state courts have prompted the water board to rescind administration of the allotments and curtailments. Water authority in California is decentralized and fragmented. Within California there are ten regional water quality control boards which exercise rulemaking and regulatory activity by watershed; there are an estimated 3,000 water service providers, 1,100 wastewater entities, 600 irrigation districts, 140 reclamation districts and 60 flood control agencies (ACWA 2014). **Fig.6** displays the extensive conveyance, storage and institutions across the State that make California’s water market a possibility.

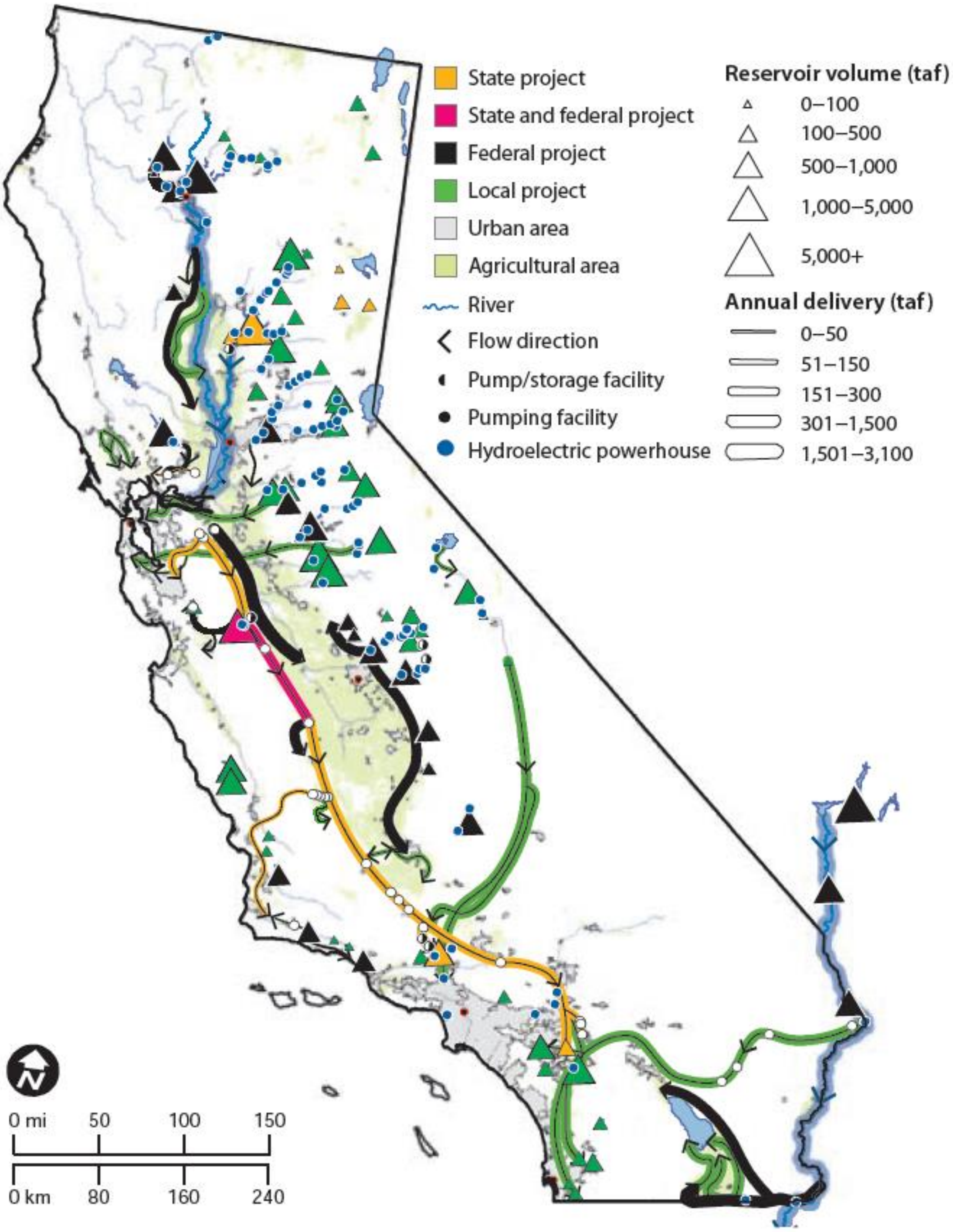


Figure 6. Inter and intrastate water infrastructure including storage and conveyance as well as capacities for annual delivery. Source, Hanak et al. 2011.

4.5 Water markets, transfers and groundwater banking

California’s water market allows for the trading of water, water traders can buy and sell water as well as short-term, long-term and permanent sales of water rights. Sellers must have the rights to use the water as well as “wet water”, the physical volume of water, and cannot trade merely a right or allocation. Buyers and sellers must be physically connected from source to destination with the conveyance infrastructure necessary for the transfer. One specific type of infrastructure includes the use of exchanges. Local authorities own the exchange and take the seller’s water and transfer the equivalent amount of water to the buyer from the exchange, thus enabling direct transfers between users in different water authorities and regions.

These water transfers allow for a more economically efficient water use (Hanak 2011). The water market was initially formed after drought throughout the 1980’s and 1990s. The market incentivizes larger suppliers, with lower value uses to transfer water to users with lower supply and higher value. Short term transfers, generally periods of a year or less, provide water during drought or during water shortages. Longer term and permanent transfers enable transitions in economic activity and consumption when demand patterns change. Currently water sales account for 3% of all water in usage with most of the trading occurring within the same region (41%) or county (38%) (Hanak 2012). Most water sales are from the farm sector, as farmers own four times the amount of water rights as compared to municipalities. Water purchases are also used to support wildlife refuges, increase water flow for fish and to reduce salt build up in the Salton Sea (Hanak 2016b).

There are many problems that arise with water transfers. Infrastructural weaknesses, such as a limited amount of conveyance systems. There is currently more demand for transfers than conveyance and exchange infrastructure in place can withstand. This leads to a limit in the market’s ability to furnish the dry-water supplies as well as limiting the availability of wet-year water to replenish groundwater banks.

The law and rules are inconsistent, laws can be complicated and specific to a location. Water trades must be approved by authorities and different rules apply for different types of water rights and authorities. Authorities are fragmented, water rights and authorities are divided over the extent of the State, with different levels of users having more senior rights which can override local authorities’ allocation (Hanak 2016a). Approval for water transfers vary depending on the county and water authority. The State Water Resources Control Board

(SWRCB) approves any transfers involving surface water rights established after 1914. Water projects approval transfers of surface water within the scope of their own projects. For example, the Central Valley Project and SWP do not need authorization from the SWRCB but do need approval from within their own projects. Water transfers involving water rights appropriated pre-1914 are not subject to approval from the SWRCB because the board does not have regulatory jurisdiction over older water rights. Water transfer that use state or federal storage or conveyance facilities are required to post a public notice and be reviewed under the California Environmental Quality Act and the National Environmental Protection Act. Regulation depends on locale; some counties do not allow the transfer of groundwater. There is no unifying law that prioritizes allocation based on a specific demand.

4.6 Ground water banking:

Groundwater banking is another mechanism that can be used to increase water supply and efficiency. Groundwater banking is the storage of water during wet years in underground aquifers used with a financial perspective in regards to withdrawals and recharges used to fulfil a water budget for a set aquifer. Infrastructure is important and necessary; pump stations and recharge facilities are required to extract groundwater and recharge water as well as the appropriate sized conveyance for transport. Aquifers that act as natural storage have a relatively large capacity and already exist in urban and agricultural areas. Due to groundwater overdraft, many aquifers have had their capacity considerably increased (Hanak 2011). Considering the improbability of the construction of new large scale surface water reservoirs, groundwater banking could be considered as a cost-effective expansion of water storage. Groundwater substitution transfers can also be considered as a form of water transfer.

5. Discussion

This chapter first presents a summary of relevant models and their conclusions. Following are a discussion of model limitations, as well as potential opportunities for meeting future restricted water supply with desalination and water demands by the import of virtual water.

5.1 Relevant Models

Vaux et al. 1984 was the first to develop an interregional and intertemporal trade model to assess the impact of water trade on California's water scarcity. Vaux' model divided California into five demand regions and eight supply regions assuming costless transportation. Vaux' model included both surface and groundwater sources as well as different demand relations for each type of region: northern agriculture, southern agriculture, Imperial Valley, northern municipal and industrial, southern municipal and industrial. Vaux introduced a curvilinear demand relationship to improve estimation of demands for irrigated water, with regional supply and demand functions for the periods of 1980, 1995 and 2020. Vaux concluded that water transfers could substitute for new supplies of water with net user benefit of $66 million in 1980 to a rise of $219 million by 2020.

Jenkins et al. 2001 presented the first comprehensive economic-engineering computer model, that incorporated statewide surface water and groundwater storage while simultaneously managing water demands. The model, the California Value Integrated Network, CALVIN, minimizes the economic operating and scarcity costs of water supply w.r.t. balance, capacity and environmental constraints to maximize statewide net economic benefit. Complexity of CALVIN is notable, both spatially and intertemporally. CALVIN includes most of California's population (92%) as well as 51 surface reservoirs, 28 groundwater basins, 19 urban economic demand areas, 24 agricultural economic demand areas, 39 environmental flow locations, 113 surface and groundwater inflows and numerous other links representing statewide conveyance. The model can be run for the 72-year period of historical inflows or modified to be run for different scenarios. Jenkins et al. concluded that "regional and statewide water markets, transfers and exchanges have a great potential to improve the flexibility and economic performance of the SWP as well as considerably reducing both water scarcity and scarcity costs".

Howitt et al. 2006 used the CALVIN model with inputs from two climate warming scenarios and one historical scenario with population and land use estimates for the year 2100. Also notable was the addition of groundwater, as it represents most annual water storage and new water supply technologies such as increased urban water usage, wastewater reuse and desalinated water. Climate warming effects were represented in all hydrologic inputs. Other changes included population-induced water demands, with an increase of state population to 92 million, as well projected increased water demands from urban and irrigation services. Costs for desalinated water were represented at $1,400 per acre foot, well under the reported costs of the more than $2,000 per acre feet. Howitt found that despite major long-term changes, California's water supply system could sustain projected increased future demands from population and climate proper management and operation of groundwater storage and significant interregional water transfers as well as added capacity from new water supply technologies.

5.2 Limitations

The model assumes a uniform marginal cost of extraction and desalination. Real world freshwater and groundwater supplies have varying per unit costs depending on annual precipitation. Freshwater extraction costs are as variable as the annual precipitation as well as the associated management and capital costs. Groundwater extraction costs rely on the capital required for extraction and annual recharge rate of the aquifer. Assuming constant annual precipitation, freshwater extraction could be considered less variable than the cost of desalination, as desalination relies on the cost of energy per unit water desalinated (Cooley 2006). Constant annual precipitation rates, or rather, in this model annually decreasing precipitation rates are improbable year after year and uncertainty in the hydrologic system is unavoidable (Kelmes, 2000a, b).

Values assumed in the model are ball park figures that are based off other average approximations. The marginal cost and desalination cost is assumed to be constant and does not take account any future technological change which may decrease the extraction cost and desalination cost.

The model also assumes that each user requires the same amount of water at each period. This is a gross simplification and depending on the degree of industrialization, individuals may increase in demand due to increased water service supply or decrease in demand due to technological increase in efficiency or better education. The model also assumes a uniform benefit function although there are varying types of consumers that have different demands and need to be satisfied depending on volume of water consumed. For example, there could be multiple consumer’s types: an industrialist would consume more water than a farmer, who would consume more than suburban user, who would, in turn consume more than an urban user. Real world water municipalities have rates for the different consumer groups. California’s farmers experienced a profound challenge when adapting to the recent drought – some farmers stopped planting and started selling water (Barringer 2011), others stopped watering their plants to harvest a different quality crop (Bland 2013), whereas others have innovated using dry farming techniques (Simmonds 2016).

The model assumes that population is a constant however different rates of population change may impact the outcome of the model. The reality is that as population increases, demand for freshwater resources will also increase. This also relates to increased demand of types of user i.e. if an agricultural region increases employment it would lead to increased water demand from that type of consumer.

This model does not take into consideration the impact of water usage on the environment i.e. there are non-market damages associated with the usage and production of water. Valuation of future environmental impacts are difficult to calculate and direct market effects which are more likely to be considered. For example, the opportunity cost of the death of a bird and the loss of its habitat are difficult to calculate and will remain difficult unless assigned an arbitrary dollar value. Ecosystem services generally provide for non-market services such as clean air and water but are generally considered as a public good. Property rights can be considered as one such mechanism of valuation although tend to not be clearly defined. Discussing water rights in California would be beyond the scope of this thesis.

The model also does not take into consideration the distribution of water availability and assumes equal access to supply. Thus, the total benefit of consumed water will not only depend on the total volume consumed, but by who and where the water is consumed.

5.3 Future opportunities:

Policy can be used to influence consumption; historic decreases in per capita water consumption can be attributed to state policies that have mandated increases in water efficiency. Technological advancement in the form of both policy adaptation, i.e. using new forms of policy such as integrated water management, and retrofitting of existing technologies and infrastructures for more water efficient usage, have decreased per capita annual water usage even during one of the most severe droughts (CDWR 2015). Hanak (2005) found that using different water use-efficiency options, such as policies, technologies and legal and institutional decision-making could reduce future water consumption by up to 20% in 2030. This results assumes that agriculture will still be a net exporter of crops globally and be responsible for the majority of interstate water demand.

One unlikely policy outcome would be to expand the total volume of surface water storage to increase supply and buffer for future demand. However, this is unlikely due to the lack of political support and the costly nature of large scale water projects.

With the likelihood of drought and increased water demand a near certainty for future California, what other opportunities exist to provide for a supply already stretched thin?

Several policy measures including but not limited to water markets, transfers and exchanges have been modeled to improve flexibility and economic performance of California’s water system even when major changes, such as population and climate warming increased demand for water. Previous studies (Vaux et al. 1984, Jenkins et al. 2001, Howitt et al. 2006) found that even historic water supplies were enough to meet demands given the ability to efficiently allocate water resources with proper conveyance and storage. Howitt et al. 2006 found that even under dramatic change i.e. water demand tripled due to increased population and climate warming, that current infrastructure in California could manage future demand.

However, assuming institutional, physical, legal and political frictions remain constant, demand will increase due to population and agricultural needs and supply will decrease due to the impacts of climate change. One option to address future supply considerations could be desalination, while another option would be to meet demand through importation of water intensive crops with the use of virtual water trade.

5.4 Desalination

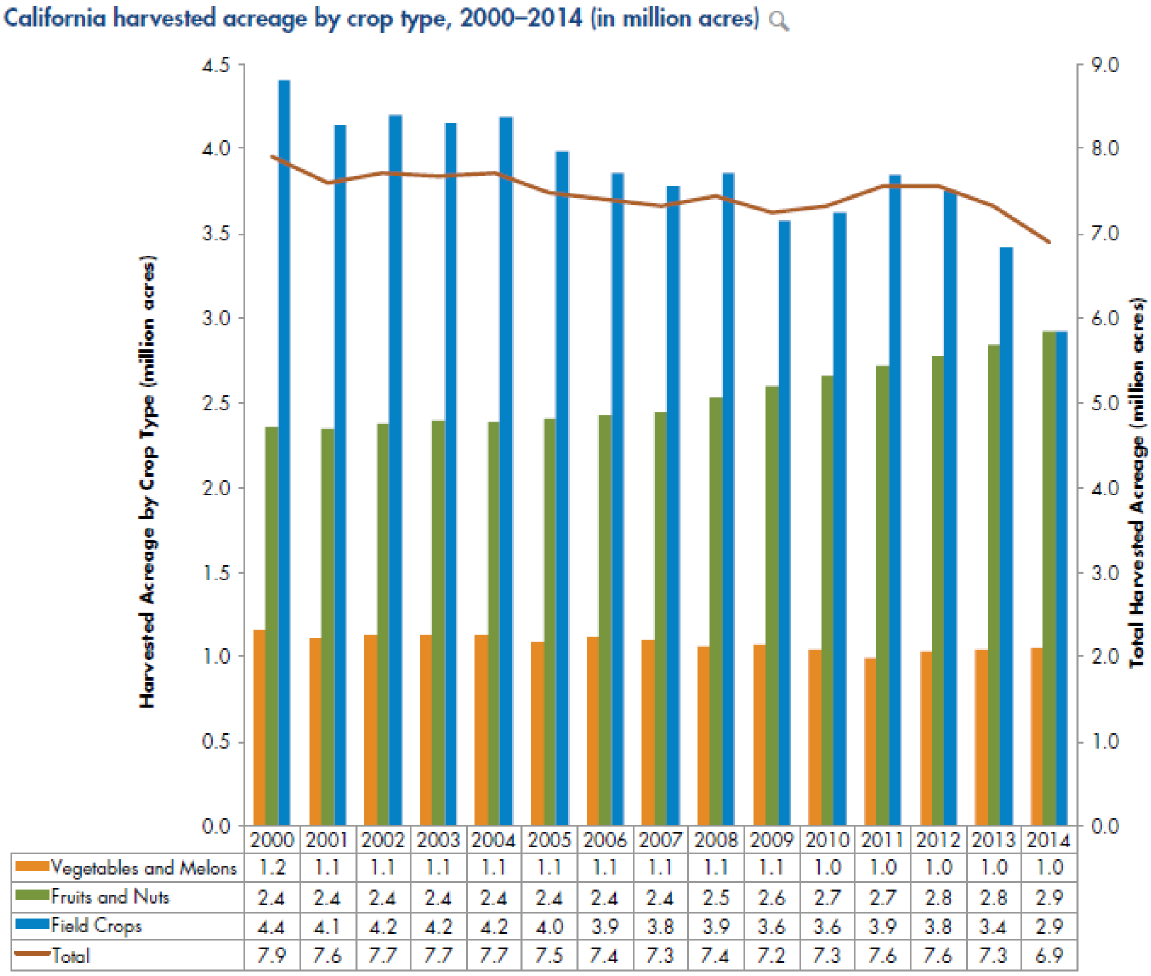
Results from literature (Jenkins et al. 2001, Howitt et al. 2006) discuss the potential for water management to increase supply through desalinated water. Desalination should be considered when all potential policy and infrastructure improvements have been exhausted. The current barriers to desalination are rather high due to its high economic costs. Other sources of supply can be found through improved wastewater reuse and more efficient use and management. Per unit costs of desalinated water are magnitudes higher than freshwater delivered from natural sources are require extensive planning and brackish effluent can impact local water quality and ecosystem services. Promising technologies such as solar desalination can potentially supply water to as low as $450 per acre-foot (WaterFX 2014), with minimal environmental impact with the requirement sited in areas with readily available sunlight. Desalination may not completely replace a total supply but it may be considered as part of the water budget in arid and water scarce municipalities that have exhausted their options and decide that they are water constrained. Results from this model show that if a region is completed water depleted, i.e. there is no longer any precipitation or water stock left, then desalination would become another source for freshwater.

5.5 Virtual Water Trade:

Virtual water (VW) is understood as the water content that has been used to produce a commodity or good and thus embedded in it (Hoekstra 2011). Originally proposed by Tony Allan in 1996 to explain why the Middle East and North African (MENA) countries did not engage in war over water resources. Allan proposed that MENA countries, which represented the first major region in the world to have "run out of water" and were dependent on regional water supplies, would become increasingly dependent on global freshwater resources. Allan argued that global soil water balanced the water budgets of MENA countries and that future demand, driven primarily by population increase, would be further balanced by global freshwater resources. Global freshwater was delivered to the MENA countries by international trade in the form of staple crops i.e. virtual water transfers (VWT) primarily through wheat.

This claim has also been shown in Spanish grain trade (Novo et. Al 2009). Novo et al (2009) analyzed the role of climate in respect to virtual transfers and water scarcity. Using specific years as measures of wet, medium and dry years to calculate the variability of virtual water trade as well as to test whether virtual water trades were more likely to occur in dry years. As predicted the dry years had substantially increased (240%) net virtual water imports when compared to wet years.

One option could be for California to transition its crop plantings, using its water resources to invest in capital intensive and higher value products such fruits and nuts. One such impact from the recent drought has been a decrease in production of field crops while higher value fruits and nuts continued to experience an increase in both harvested area and revenue (Cooley 2015).


California harvested acreage by crop type, 2000–2014 (in million acres)

	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
🟧 Vegetables and Melons	1.2	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.0	1.0	1.0	1.0	1.0
🟩 Fruits and Nuts	2.4	2.4	2.4	2.4	2.4	2.4	2.4	2.4	2.5	2.6	2.7	2.7	2.8	2.8	2.9
🟦 Field Crops	4.4	4.1	4.2	4.2	4.2	4.0	3.9	3.8	3.9	3.6	3.6	3.9	3.8	3.4	2.9
— Total	7.9	7.6	7.7	7.7	7.7	7.5	7.4	7.3	7.4	7.2	7.3	7.6	7.6	7.3	6.9

Figure 7. Harvest acreage by crop type from 2000-2014. Source: Cooley 2015.

Fig. 6 illustrates the recent trends from the harvested area showing a decreased in total harvested acreage due to increased water cost, while at the same time an increased proportion of higher value crops were planted. Similarly, a water scarce future would require similar agricultural adaptations to meet a decreased irrigation water supply. The idea of continuing this trend in a future with limited precipitation may help offset any loss of revenue from a decrease in available irrigation water as well as make up any deficient from the importation of lower value staple crops that require a higher volume of water to produce.

6. Conclusion

This thesis considers an economic model to firstly, represent and analyze a social planner maximization problem, and secondly suggest an optimal extraction strategy when a backstop technology is present and finally to develop conclusions for ideas to meet the potential challenges incurred through water scarcity.

The model offers insight into optimal extraction during periods of decreasing freshwater resources with a backstop technology. The social planner implies the maximized discounted sum of benefits (net of costs) over an infinite time horizon. At a certain point, freshwater influx will no longer be able to sustain the stock of freshwater and thus the benefit from freshwater is derived from the production of desalinated water.

Some locations in the world already require desalination to provide for their societies' freshwater budget. Desalination could be an option for supplying arid and water scarce locations in which the water supply is completely constrained, however all options must be considered due to the prohibitively high initial investment into a desalination plant. Other ways to increase supply such as relatively low budget investments into retrofits and high efficiency infrastructure and policy measures should be prioritized before committing to desalination. In locations where agricultural productivity remains high and can produce higher value crops but water remains scare, virtual water trade could be considered to balance trades of embedded water.

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8. Appendix A: Renewable Resource Model

Social Planner Problem:

$$\underset{y_t, z_t}{\text{Max}} \sum_{t=0}^{\infty} [N \cdot B\left(\frac{y_t + z_t}{N}\right) - \alpha y_t - \frac{\beta}{2} z_t^2] \delta^t$$

$$\text{s.t } x_{t+1} = x_t - y_t + \varepsilon_t \text{ , } x_0 \text{ given and } \varepsilon_0 \text{ given}$$

With $\varepsilon_t = \varepsilon_0 \cdot {R_\varepsilon}^t$ and the benefit function $B(w_t)$=$\left(\frac{y_t+z_t}{N}\right)$, equal to a quadratic $B(w_t) = \Gamma w_t(\Phi - \frac{1}{2} w_t)$. And *N*, representing a constant population or individual; each *N*, will demand an identical amount of water in each period, *t*. To solve intertemporal maximization problem, we will use Lagrangian multiplier to maximize the objectives in every period *t*, in this case the stock of water x_t, freshwater extraction, y_t and freshwater influx, ε_t as well as to minimize the constraints of marginal cost of fresh water extraction α and the marginal cost of desalination β.

Lagrangian:

$$L = \sum_{t=0}^{\infty} \delta^t \left[N \cdot B\left(\frac{y_t + z_t}{N}\right) - \alpha y_t - \frac{\beta}{2} zt^2 + \delta \lambda_{t+1}(x_t - y_t + \varepsilon_t - x_{t+1})\right]$$

We obtain the first-order conditions by differentiating with respect to y_t, z_t and x_t.

First Order Conditions:

1. $\frac{\partial L}{\partial y_t} = \delta^t \left[N \cdot B'\left(\frac{y_t+z_t}{N}\right) \cdot \frac{1}{N} - \alpha - \delta \lambda_{t+1}\right] = 0$
2. $\frac{\partial L}{\partial z_t} = \delta^t \left[N \cdot B'\left(\frac{y_t+z_t}{N}\right) \cdot \frac{1}{N} - \beta z_t\right] = 0$
3. $\frac{\partial L}{\partial x_t} = \delta^t [\delta \lambda_{t+1} - \lambda_t] = 0$

Following derivation of the FOCs:

1. $B'^{\left(\frac{y_t+z_t}{N}\right)} = \alpha + \delta \lambda_{t+1}$
2. $B'^{\left(\frac{y_t+z_t}{N}\right)} = \beta z_t$

Set $B\left(\frac{y_t+z_t}{N}\right) = \Gamma w \left(\Phi - \frac{1}{2} w\right) \rightarrow B'\left(\frac{y_t+z_t}{N}\right) = \Gamma(\Phi - w)$

Which follows that:

$$\Gamma(\Phi - w) = \beta z_t$$

$$N\Gamma\Phi - \Gamma y_t - \Gamma z_t = N\beta z_t$$

$$z_t(N\beta + \Gamma) = \Gamma(N\Phi - y_t)$$

Which gives us the solution for z_t:

$$z_t = \frac{\Gamma(N\Phi - y_t)}{N\beta + \Gamma}$$

Following we can find y_t:

$$\Gamma y_t = N\Gamma\Phi - (N\Phi\beta + \Gamma)z_t$$

$$y_t = N\Phi - \frac{N\Phi\beta + \Gamma}{\Gamma} z_t$$

Inserting the second equation into the first, we will arrive at:

$$\beta z_t = \alpha + \delta\lambda_{t+1} \to \delta\lambda_{t+1} = \beta z_t - \alpha$$

Algebraically this leads to:

$$\delta\lambda_t = \beta z_t - \alpha \to \lambda_t = \frac{\beta}{\delta} z_{t-1} - \frac{\alpha}{\beta}$$

Which simplifies to:

$$\beta z_{t+1} - \alpha = \frac{\beta}{\delta} z_{t-1} - \frac{\alpha}{\beta}$$

$$z_{t+1} = \frac{\alpha}{\beta} - \frac{\beta}{\beta\delta} + \frac{1}{\delta} z_t$$

Thus, the solution for z_{t+1} is:

$$z_{t+1} = \frac{1}{\delta} z_t + \frac{\alpha}{\beta}(1 - \frac{1}{\delta})$$

Recalling that $y_t = N\Phi - \frac{N\Phi\beta + \Gamma}{\Gamma} z_t$, we can plug the solution for y_t into a system of two difference equations:

1) $z_{t+1} = \frac{1}{\delta} z_t + \frac{\alpha}{\beta}\ (1 - \frac{1}{\delta})$

2) $x_{t+1} = x_t - N\Phi + (\frac{N\Phi\beta+\Gamma}{\Gamma})z_t + \varepsilon_t$

Solving for z_t, by constraining for y_t will result in the following solution:

$$z_t = z_0\delta^{-t} + \frac{\alpha}{\beta}(1-\delta^{-t}) = \frac{\alpha}{\beta} + \left(z_0 - \frac{\alpha}{\beta}\right)\,\delta^{-t}$$

We can then test to see if this does in fact hold, and it does:

$$z_0 \cdot \delta^{-(t+1)} + \frac{\alpha}{\beta}\left(1-\delta^{-(t+1)}\right) = \frac{1}{\delta}\left[z_0\delta^{-t} + \frac{\alpha}{\beta}(1-\delta^{-t})\right] + \frac{\alpha}{\beta}(1-\frac{1}{\delta})$$

Then we can plug the solution for z_t into the difference equation for x_{t+1}:

$$x_{t+1} = x_t - N\Phi + \left(\frac{N\Phi\beta+\Gamma}{\Gamma}\right)\left[\frac{\alpha}{\beta} + \left(z_0 - \frac{\alpha}{\beta}\right)\delta^{-t}\right] + \varepsilon_t$$

As a reminder, $\varepsilon_t = \varepsilon_0 \cdot {R_\varepsilon}^t$. Thus:

$$x_{t+1} = x_t - N\Phi + \frac{N\Phi\beta+\Gamma}{\Gamma}\cdot\frac{\alpha}{\beta} + \left(\frac{N\Phi\beta+\Gamma}{\Gamma}\right)\left(z_0 - \frac{\alpha}{\beta}\right)\delta^{-t} + \varepsilon_0 \cdot {R_\varepsilon}^t$$

$$x_{t+1} = x_t + \left(N\frac{\beta}{\Gamma}+1\right)\left(z_0 - \frac{\alpha}{\beta}\right)\delta^{-t} + \varepsilon_0 \cdot {R_\varepsilon}^t + \left(N\frac{\beta}{\Gamma}+1\right) - N\Phi$$

Solving for x_t, we receive an inhomogeneous solution for x_t:

$$x_t = A_1 \cdot \delta^{-t} + A_2 \cdot {R_\varepsilon}^t + A_3 \cdot t$$

$$\begin{aligned}
&A_1 \cdot \delta^{-(t+1)} + A_2 \cdot {R_\varepsilon}^{t+1} + A_3(t+1) \\
&\qquad = A_1 \cdot \delta^{-t} + A_2 \cdot {R_\varepsilon}^t + A_3 \cdot t + \left(N\frac{\beta}{\Gamma}+1\right)\left(z_0 - \frac{\alpha}{\beta}\right)\delta^{-t} + \varepsilon_0 \cdot {R_\varepsilon}^t \\
&\qquad + \left(N\frac{\Gamma}{\beta}+1\right)\frac{\alpha}{\beta} - N\Phi
\end{aligned}$$

$$\delta^{-t}\left[\frac{A_1}{\delta} - A_1 - (N\frac{\beta}{\Gamma}+1)\left(z_0 - \frac{\alpha}{\beta}\right)\right] + {R_\varepsilon}^t[A_2R_\varepsilon - A_2 - \varepsilon_0] + \ A_3 - \left(N\frac{\beta}{\Gamma}+1\right)\frac{\alpha}{\beta} + N\Phi = 0$$

Setting up for a recurrence relation with A_1:

$$A_1\left(\frac{1}{\delta} - 1\right) = (N\frac{\beta}{\Gamma}+1)\left(z_0 - \frac{\alpha}{\beta}\right)$$

Simplified to:

$$A_1 = \frac{\delta}{1-\delta}(N\frac{\beta}{\Gamma}+1)\left(z_0 - \frac{\alpha}{\beta}\right)$$

With A_2:

$$A_2(R_\varepsilon - 1) - \varepsilon_0 = 0$$

Simplified to:

$$A_2 = \frac{\varepsilon_0}{R_\varepsilon - 1}$$

With A_3:

$$A_3 = \left(N\frac{\beta}{\Gamma}+1\right)\frac{\alpha}{\beta} - N\Phi$$

Plugging in A_1, A_2, and A_3 leads to:

$$x_t = \frac{\delta}{1-\delta}\left(N\frac{\beta}{\Gamma}+1\right)\left(z_0 - \frac{\alpha}{\beta}\right)\cdot \delta^{-t} + \frac{\varepsilon_0}{R_\varepsilon - 1}\cdot {R_\varepsilon}^{t} + [\left(N\frac{\beta}{\Gamma}+1\right)\frac{\alpha}{\beta} - N\Phi]\cdot t$$

$$x_t = A + \frac{\delta}{1-\delta}\left(N\frac{\beta}{\Gamma}+1\right)\left(z_0 - \frac{\alpha}{\beta}\right)\cdot \delta^{-t} + \frac{\varepsilon_0}{R_\varepsilon - 1}\cdot {R_\varepsilon}^{t} + [\left(N\frac{\beta}{\Gamma}+1\right)\frac{\alpha}{\beta} - N\Phi]\cdot t$$

Isolating for x_0:

$$x_0 = A + \frac{\delta}{1-\delta}\left(N\frac{\beta}{\Gamma}+1\right)\left(z_0 - \frac{\alpha}{\beta}\right) + \frac{\varepsilon_0}{R_\varepsilon - 1}$$

Thus, we can solve for *A*:

$$A = x_0 - \frac{\delta}{1-\delta}\left(N\frac{\beta}{\Gamma}+1\right)\left(z_0 - \frac{\alpha}{\beta}\right) + \frac{\varepsilon_0}{R_\varepsilon - 1}$$

Plugging in *A* in the previous equation for x_t, we result in:

$$x_t = x_0 + \frac{\delta}{1-\delta}\left(N\frac{\beta}{\Gamma}+1\right)\left(z_0 - \frac{\alpha}{\beta}\right)(\delta^{-t} - 1) + \frac{\varepsilon_0}{R_\varepsilon - 1}\left({R_\varepsilon}^{t} - 1\right) + [\left(N\frac{\beta}{\Gamma}+1\right)\frac{\alpha}{\beta} - N\Phi] \cdot t$$

Recalling that $z_t = z_0\delta^{-t} + \frac{\alpha}{\beta}(1-\delta^{-t})$ and $y_t = N\Phi - \frac{N\Phi\beta+\Gamma}{\Gamma} z_t$, we can plug the solution of z_t into the equation for y_t:

$$y_t = N\Phi - \frac{N\Phi\beta + \Gamma}{\Gamma}\cdot [z_0\delta^{-t} + \frac{\alpha}{\beta}(1-\delta^{-t})]$$

$$y_0 = N\Phi - \frac{N\Phi\beta + \Gamma}{\Gamma} \cdot z_0$$

Then to isolate for z_0:

$$z_0 \frac{N\beta + \Gamma}{\Gamma} = N\Phi - y_0$$

$$z_0 = \frac{N\Phi - y_0}{N\Phi \frac{\beta}{\Gamma} + 1}$$

Plugging z_0 in to y_t to solve for y_t:

$$y_t = \ N\Phi - (N\Phi - y_0)\delta^{-t} - \frac{\alpha}{\beta}\frac{N\beta + \Gamma}{\Gamma}(1 - \delta^{-t})$$

Simplifying through distributive properties:

$$y_t = \ y_0\delta^{-t} + \ N\Phi(1 - \delta^{-t}) - (N\frac{\alpha}{\Gamma} + \frac{\alpha}{\beta})(1 - \delta^{-t})$$

$$y_t = \ y_0\delta^{-t} + [N\left(\Phi - \frac{\alpha}{\beta}\right) - \frac{\alpha}{\beta}](1 - \delta^{-t})$$

We can then abbreviate by introducing *K*, all terms being constant:

$$K = N\left(\Phi - \frac{\alpha}{\Gamma}\right) - \frac{\alpha}{\beta}$$

$$x_t = x_0 + \frac{\delta}{1-\delta}\left(N\Phi - y_0 - N\frac{\alpha}{\beta} - \frac{\alpha}{\beta}\right)(\delta^{-t} - 1) + \frac{\varepsilon_0}{R_\varepsilon - 1}\left({R_\varepsilon}^t - 1\right) + [N \cdot \frac{\alpha}{\Gamma} + \frac{\alpha}{\beta} - N\Phi] \cdot \mathrm{t}$$

$$x_t = x_0 - y_0\frac{\delta(\delta^{-t} - 1)}{1-\delta} + \varepsilon_0\frac{\varepsilon_0}{R_\varepsilon - 1} + K[\frac{\delta(\delta^{-t} - 1)}{1-\delta} - t]$$

$$y_t = y_0\delta^{-t} - K(\delta^{-t} - 1)$$

$$z_t = \frac{N\Phi - y_0}{N\frac{\beta}{\Gamma} + 1}\delta^{-t} + \frac{\alpha}{\beta}(1 - \delta^{-t})$$

There exists at some time, *T*, for which holds:

$$y_T = \varepsilon_T, \forall \ t \geq T$$

$$y_T = y_0 \cdot \delta^{-T} - K(\delta^{-T} - 1) = \varepsilon_0 \cdot {R_\varepsilon}^T$$

$$y_0 = \varepsilon_0 \cdot {R_\varepsilon}^T \cdot \delta^T + K(1 - \delta^T)$$

$$y_t = \varepsilon_0 \cdot {R_\varepsilon}^T \cdot \delta^{T-t} + K(\delta^{-t} - \delta^{T-t}) - K(\delta^{-t} - 1)$$

$$y_t = \varepsilon_0 \cdot {R_\varepsilon}^T \cdot \delta^{T-t} + K(1 - \delta^{T-t})$$

To solve for the period T¸ we will first solve the LHS and the RHS then which gives the solution for the model:

$$x_0 + \sum_{t=0}^{T-1} \varepsilon_t = \sum_{t=0}^{T-1} y_t$$

$$x_0 + \varepsilon_0 \sum_{t=0}^{T-1} \cdot {R_\varepsilon}^t = \sum_{t=0}^{T-1} [\varepsilon_0 \cdot {R_\varepsilon}^T \cdot \delta^{T-t} + K(1 - \delta^{T-t})]$$

$$x_0 + \varepsilon_0 \frac{{R_\varepsilon}^T - 1}{R_\varepsilon - 1} = \varepsilon_0 \cdot {R_\varepsilon}^T \cdot \delta^T \frac{\delta}{1 - \delta} (\delta^{-T} - 1) - K\delta^T \cdot \frac{\delta}{1 - \delta} (\delta^{-T-1}) + KT$$

$$x_0 + \varepsilon_0 \frac{{R_\varepsilon}^T - 1}{R_\varepsilon - 1} = \varepsilon_0 \cdot {R_\varepsilon}^T \frac{\delta}{1 - \delta} (1 - \delta^T) - K \cdot \frac{\delta}{1 - \delta} (1 - \delta^T) + KT$$

Declaration

under Art. 28 Para. 2 RSL 05

Last, first name: Ackerman, Alden

Matriculation number: 13-108-774

Programme: Masters of Science, Climate Sciences

Bachelor ☐ Master ☒ Dissertation ☐

Thesis title: Intertemporal Allocation of Water Resources

Thesis supervisor: Prof. Dr. Ralph Winkler

I hereby declare that this submission is my own work and that, to the best of my knowledge and belief, it contains no material previously published or written by another person, except where due acknowledgement has been made in the text. In accordance with academic rules and ethical conduct, I have fully cited and referenced all material and results that are not original to this work. I am well aware of the fact that, on the basis of Article 36 Paragraph 1 Letter o of the University Law of 5 September 1996, the Senate is entitled to deny the title awarded on the basis of this work if proven otherwise. I grant inspection of my thesis.

Luzern, 2/14/2017
Place, date

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Signature