On the Mass, Terminal Velocity and Kinetic Energy of Natural Hailstones from Field Observations and Laboratory Experiments

Andrew Heymsfield
NCAR, Boulder CO USA
heyms1@ucar.edu

With significant contributions from Ian Giammanco and Robert Wright

Overview

The basic idea is that at equilibrium, the gravitational force acting on a particle is resisted by its drag force

$$c_d[1/2\rho_f v_t^2 A] = mg$$

Basic equation for calculating the terminal velocity from the drag coefficient is therefore:

$$V_t = (2mg/\rho_f C_d A)^{0.5}$$

nal

area D its diameter

The Reynolds number is given by:

$$\mathrm{Re} = V_t D / v$$

where v is the kinematic viscosity of air

Kinetic energy is KE=1/2mV_t^2

Factors Contributing to Hail Size at Surface

- Large stones: better chance to reach ground than small stones which melt faster on their descent

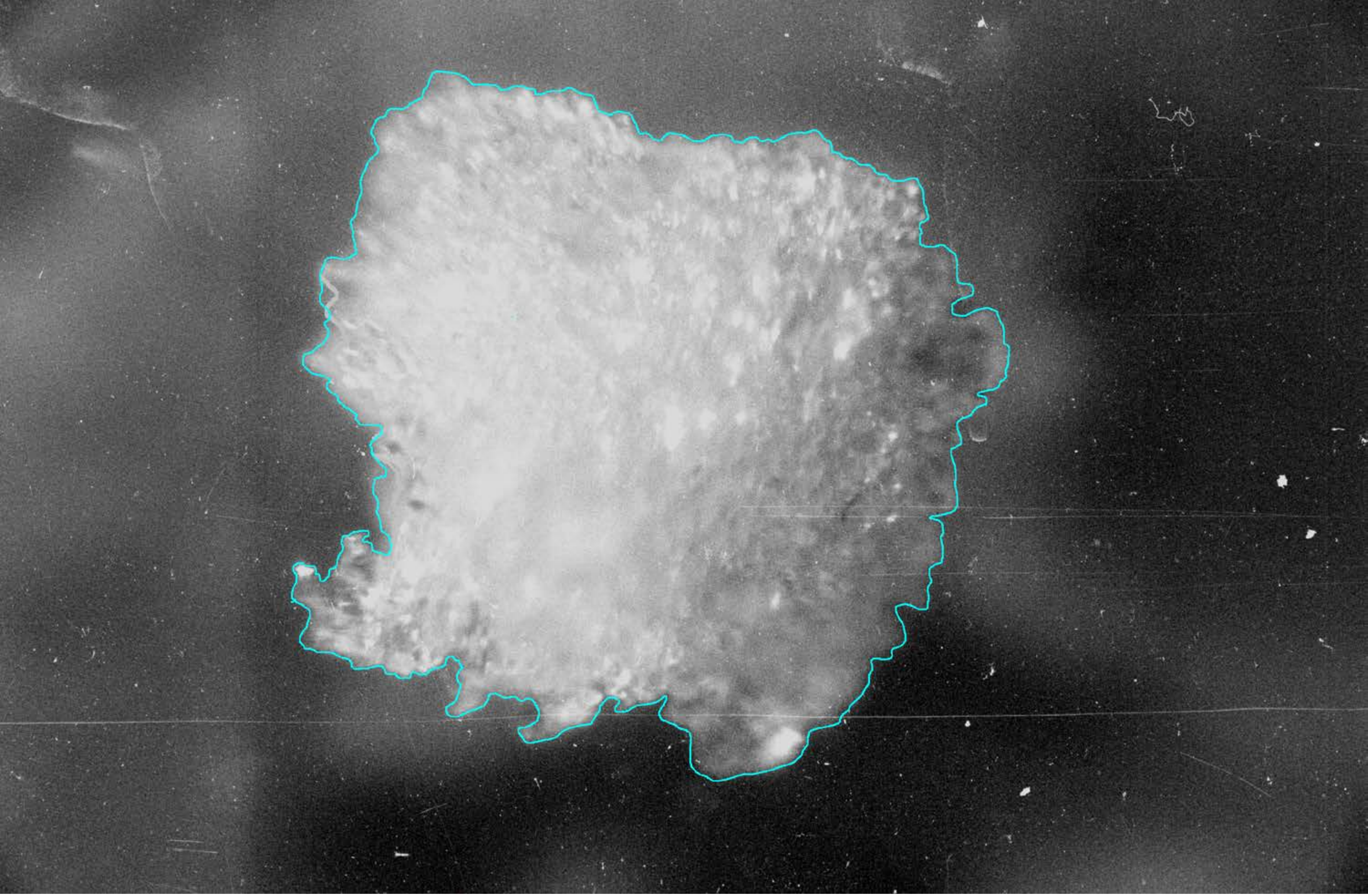
- Stones melt faster in wet environment than in dry (RH has strong effect); dry air in ~800-500 mb layer is very important to produce evaporative cooling and limit hail melting during descent (why stones are smaller or non-existent during MCSs due to saturated environment – and weaker updraft)

- Hail falling within rain melts at a much greater rate than hail falling separate from rain (especially for small hail)

- Vertical wind shear is critical in surface hail size to separate the updraft and downdraft and limit duration hail falls in heavy rain. May be more important than vertical thermal and moisture profiles.

- Greatest rate of melting: small stones in wet environments (high RH and within rain)
- Least rate of melting: very large stones (≥ golfball) in dry environments outside of heavy rain shaft, *even in warm environment with high freezing level*

- **Graupel and hail terminal velocity matters!**



Original Photograph of Graupel Particle Sampled by Locatelli and Hobbs (1974)

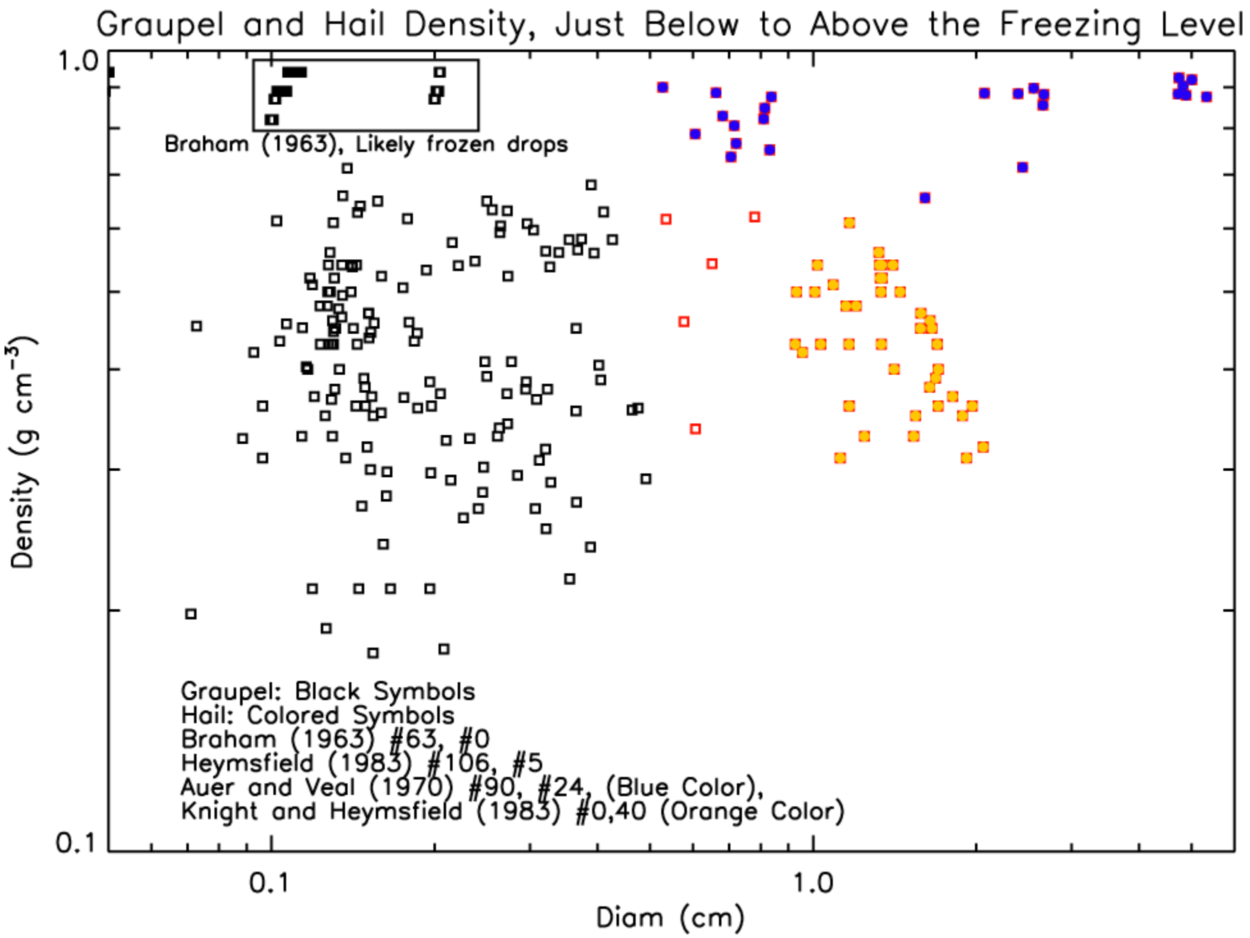
Photograph of the largest hailstone ever colleted (Vivian South Dakota, July 23 2010)

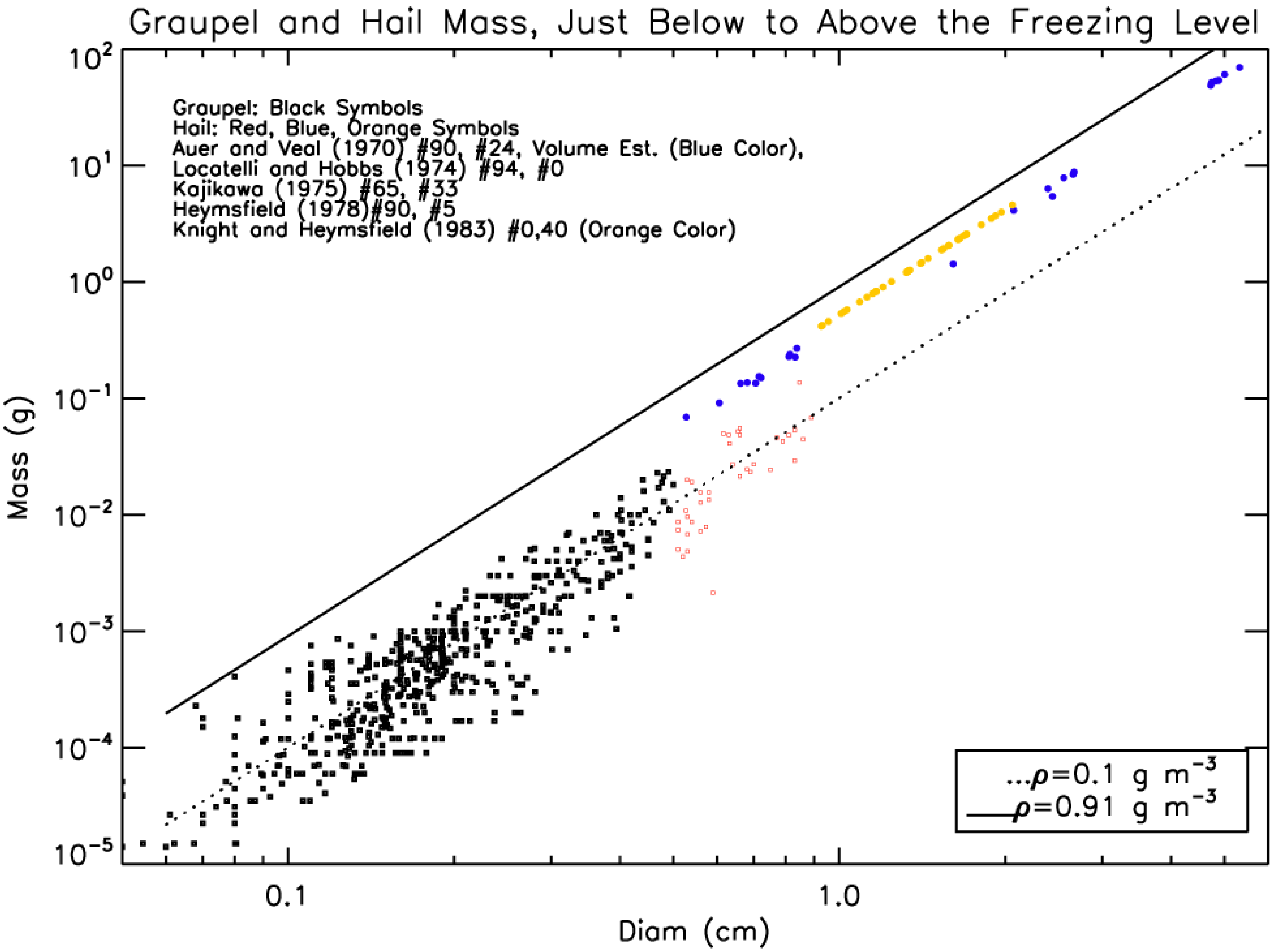
Giammanco (2017, BAMS) scanned many hailstones collected in field programs
In Nebraska, Kansas, Colorado and Texas

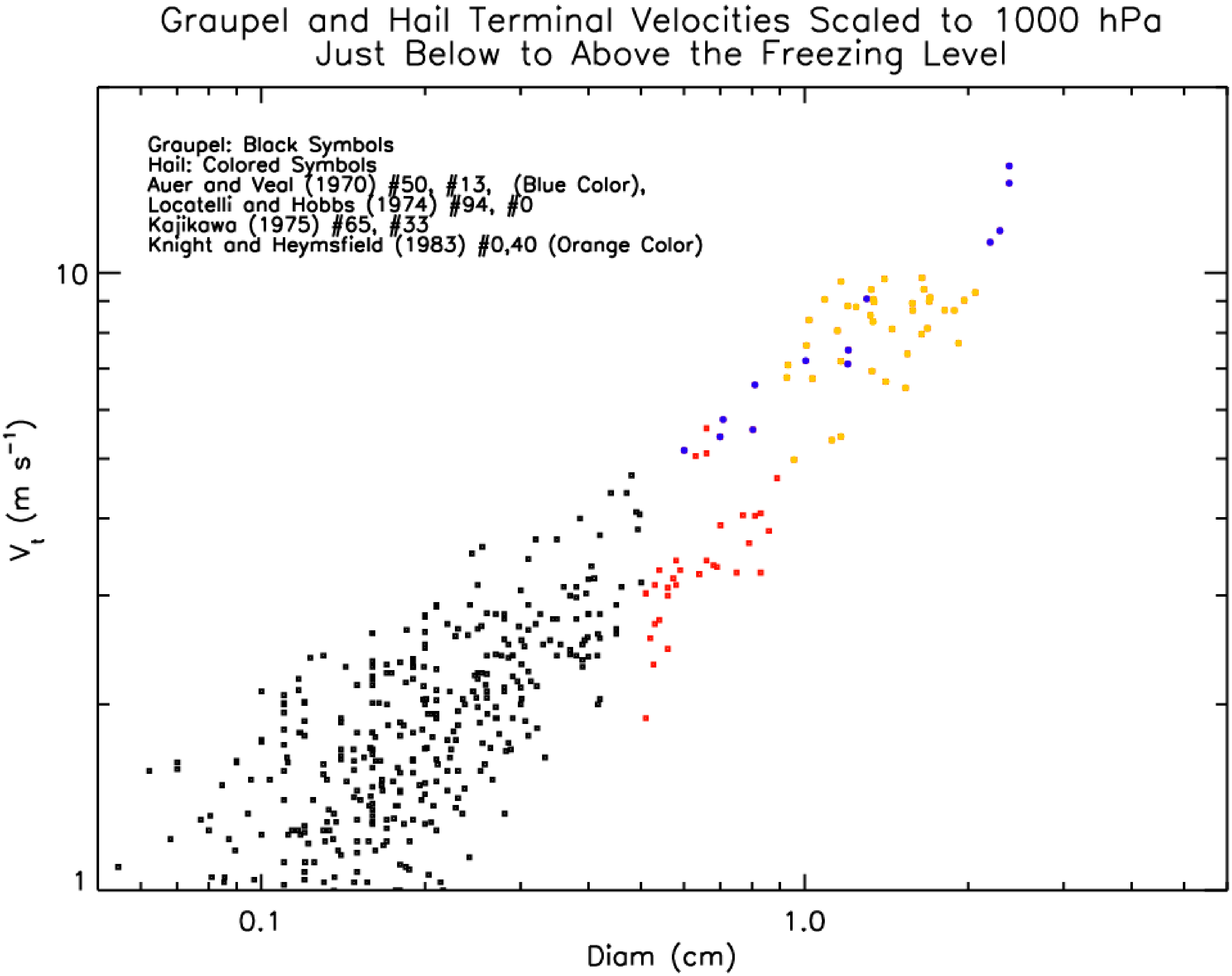
Together, he and I scanned the largest hailstones ever collected (Coffeyville, Kansas;
Aurora, Nebraska, and Vivian South Dakota

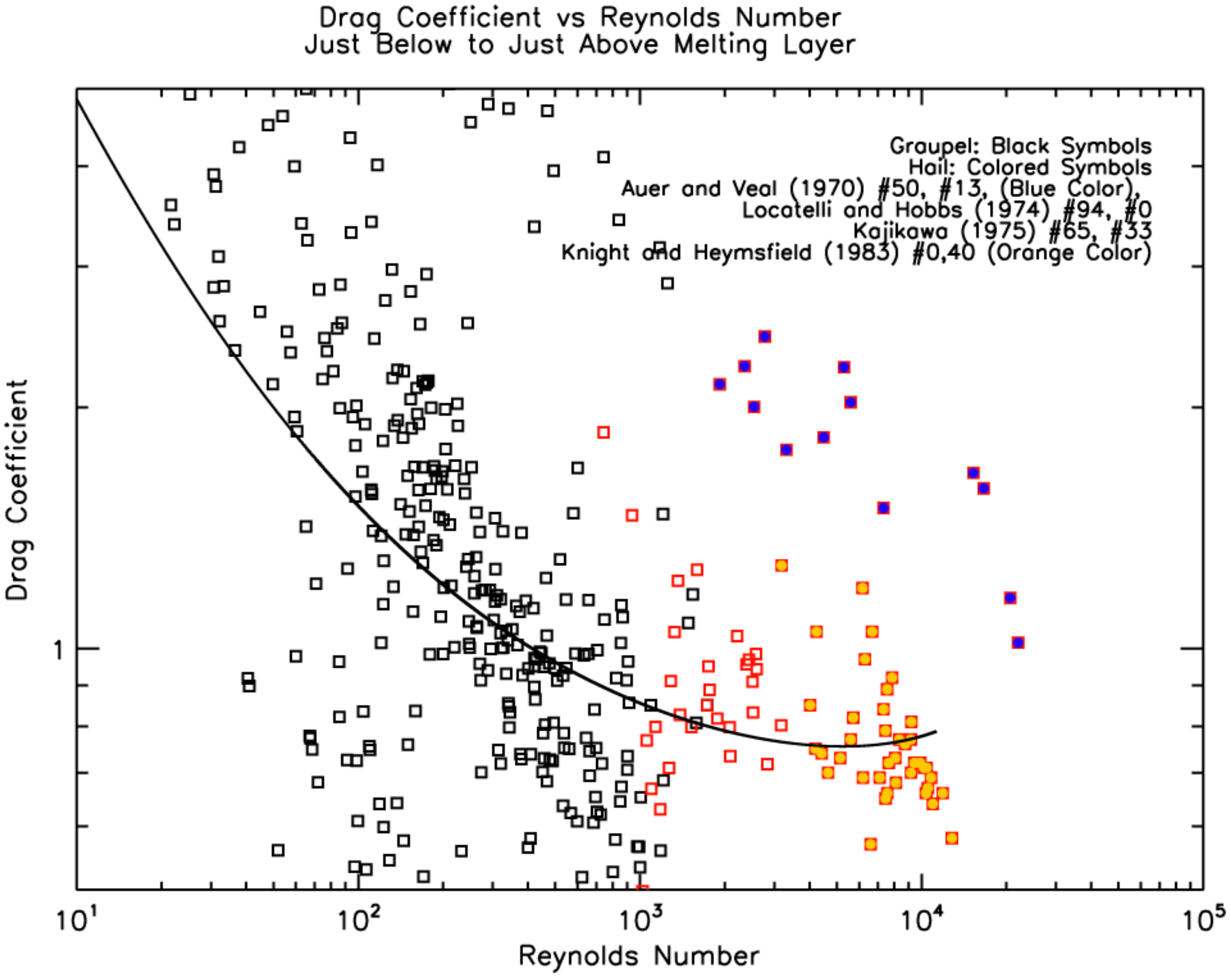
More on why this was done later in my talk

▶


Graupel and Hail Density, Just Below to Above the Freezing Level
Braham (1963), Likely frozen drops
Graupel: Black Symbols
Hail: Colored Symbols
Braham (1963) #63, #0
Heymsfield (1983) #106, #5
Auer and Veal (1970) #90, #24, (Blue Color),
Knight and Heymsfield (1983) #0,40 (Orange Color)
1.0
0.1
Density (g cm^{-3})
0.1
1.0
Diam (cm)


Graupel and Hail Mass, Just Below to Above the Freezing Level
Graupel: Black Symbols
Hail: Red, Blue, Orange Symbols
Auer and Veal (1970) #90, #24, Volume Est. (Blue Color),
Locatelli and Hobbs (1974) #94, #0
Kajikawa (1975) #65, #33
Heymsfield (1978)#90, #5
Knight and Heymsfield (1983) #0,40 (Orange Color)
...ρ=0.1 g m⁻³
ρ=0.91 g m⁻³
Mass (g)
Diam (cm)
10²
10¹
10⁰
10⁻¹
10⁻²
10⁻³
10⁻⁴
10⁻⁵
0.1
1.0


Graupel and Hail Terminal Velocities Scaled to 1000 hPa
Just Below to Above the Freezing Level
Graupel: Black Symbols
Hail: Colored Symbols
Auer and Veal (1970) #50, #13, (Blue Color),
Locatelli and Hobbs (1974) #94, #0
Kajikawa (1975) #65, #33
Knight and Heymsfield (1983) #0,40 (Orange Color)
10
1
0.1
1.0
Diam (cm)
V_t (m s^-1)



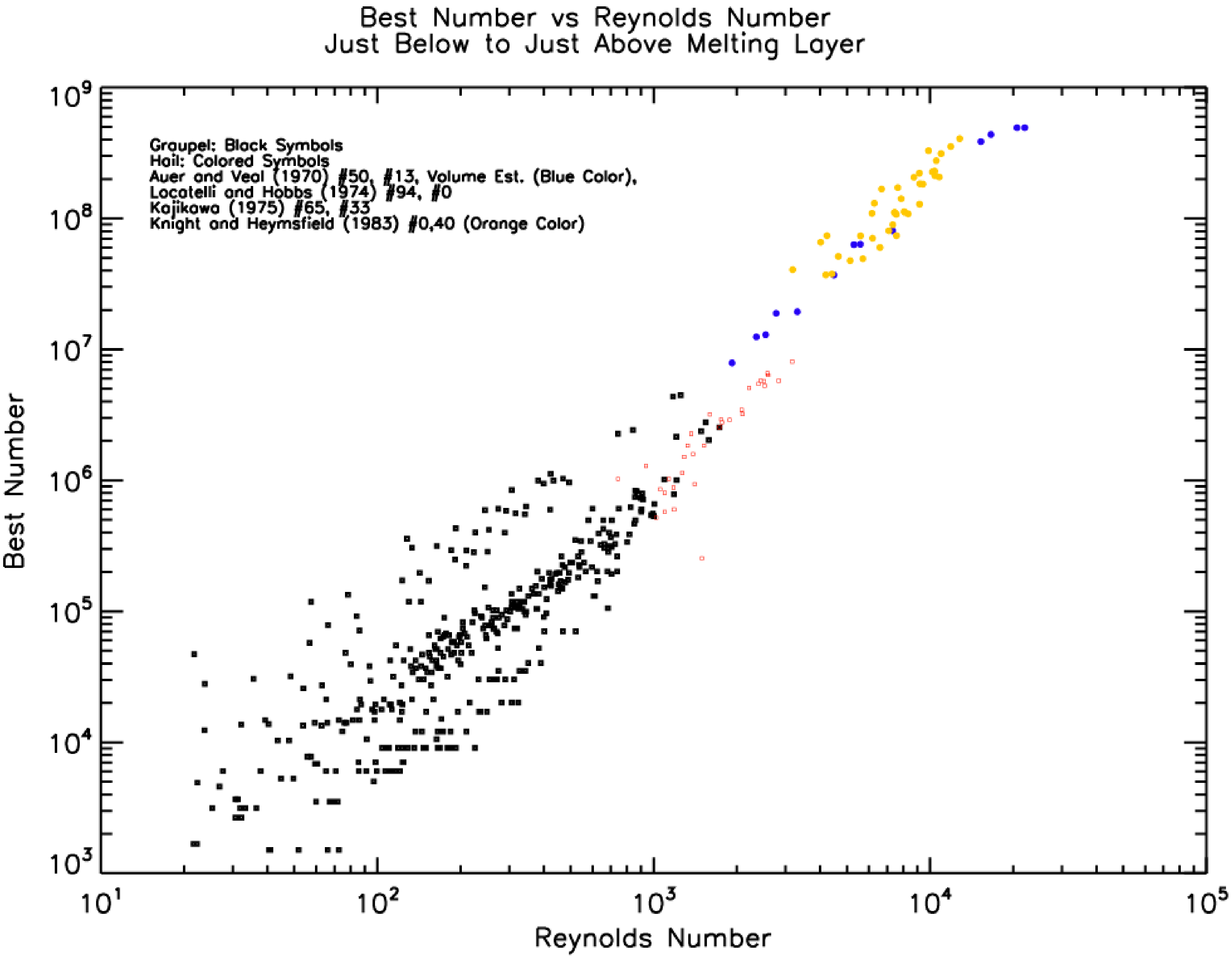
Drag Coefficient vs Reynolds Number
Just Below to Just Above Melting Layer

For calculation of terminal velocities, it is convenient to use the non-dimensional Davies or Best number (X),

$$\mathrm{X} = C_d \, \mathrm{Re}^2 \, ,$$

where

$$\mathrm{X} = 2mD^2 g / (\rho_f v^2 A)$$


Best Number vs Reynolds Number
Just Below to Just Above Melting Layer
Graupel: Black Symbols
Hail: Colored Symbols
Auer and Veal (1970) #50, #13, Volume Est. (Blue Color),
Locatelli and Hobbs (1974) #94, #0
Kajikawa (1975) #65, #33
Knight and Heymsfield (1983) #0,40 (Orange Color)
Best Number
Reynolds Number
10^9
10^8
10^7
10^6
10^5
10^4
10^3
10^1
10^2
10^3
10^4
10^5

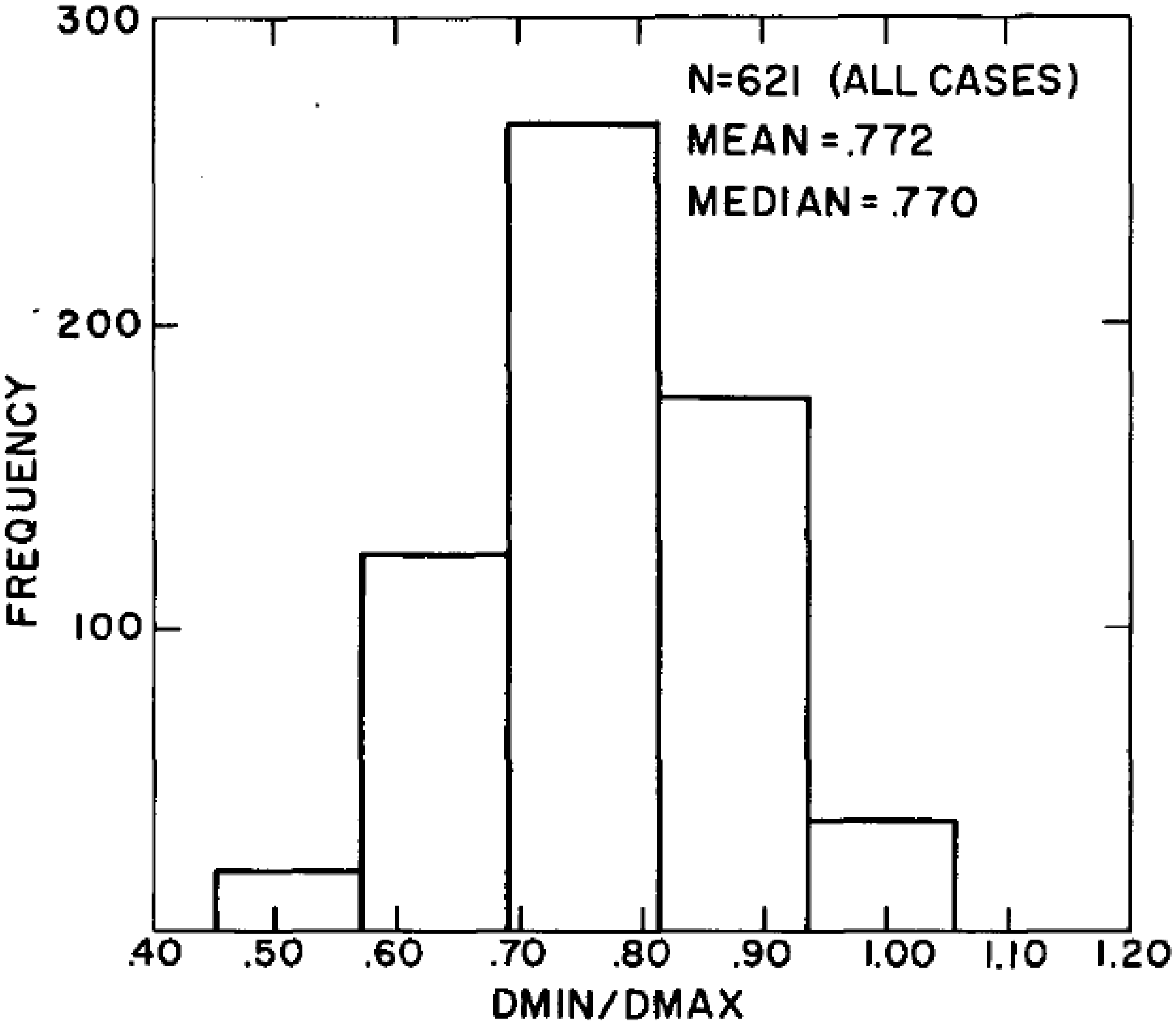
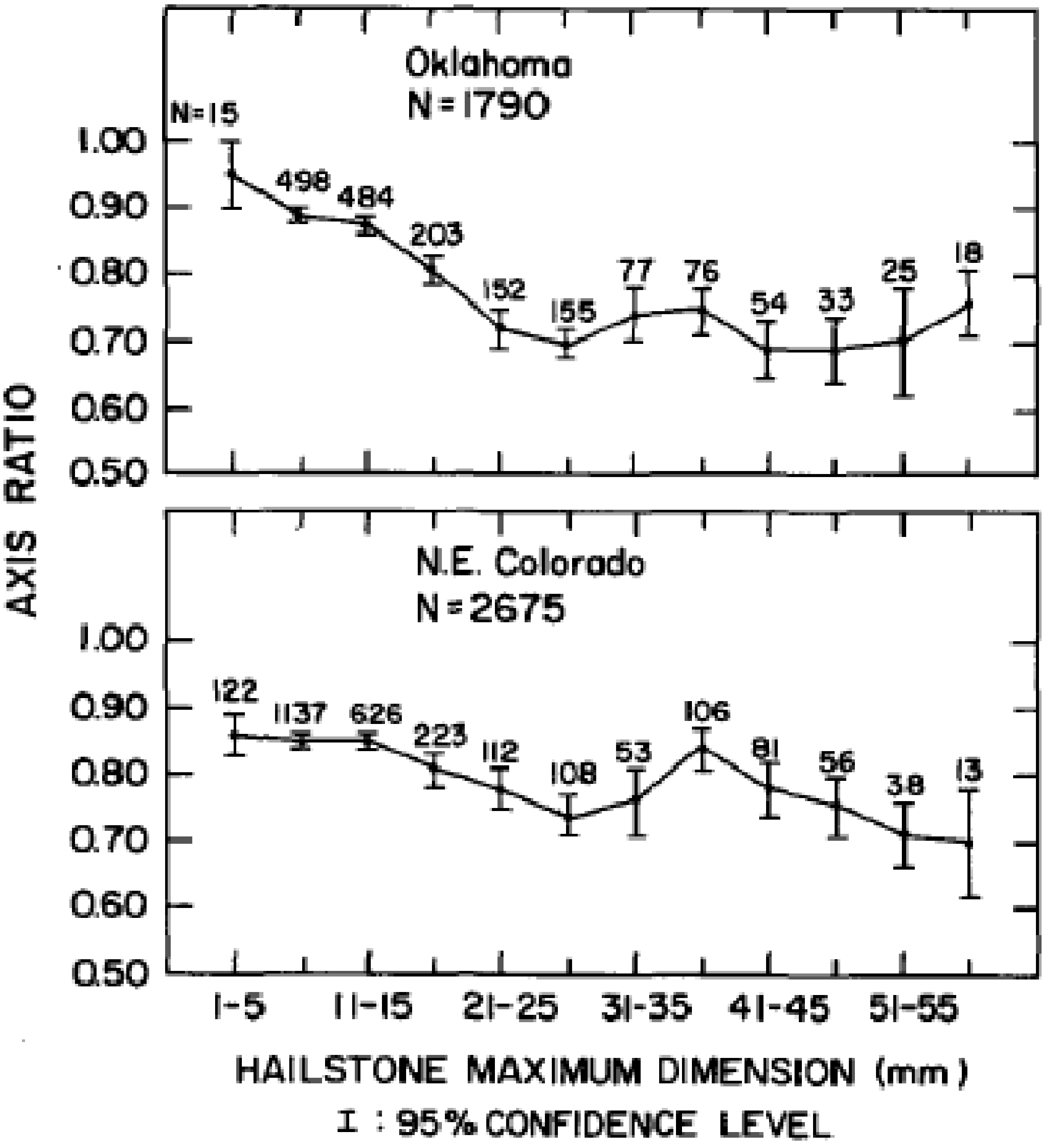


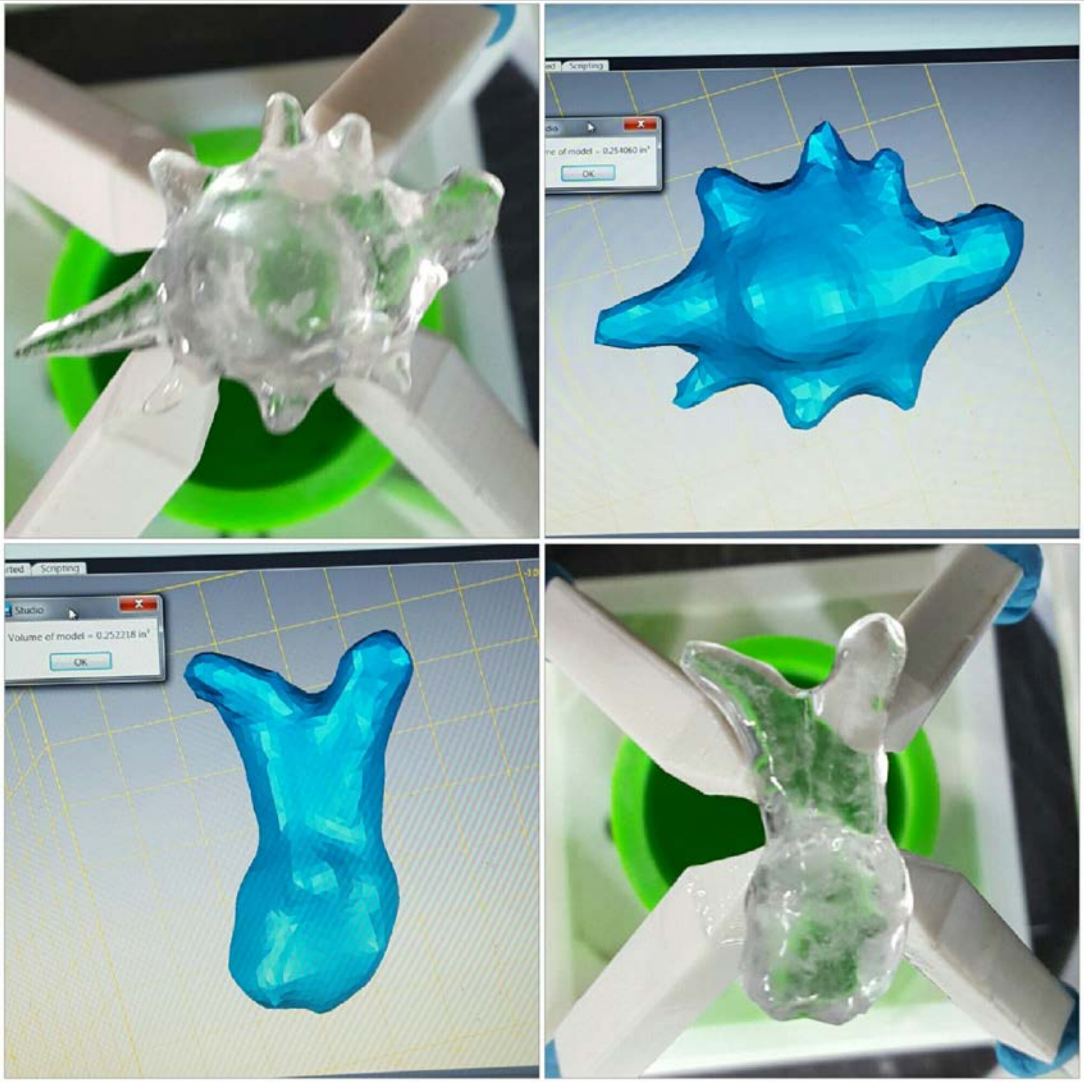
FIG. 10. Histogram of frequency versus ratio of minimum to maximum hailstone dimensions.

Matson and Huggins (1980)

Wyoming, Colorado, Nebraska

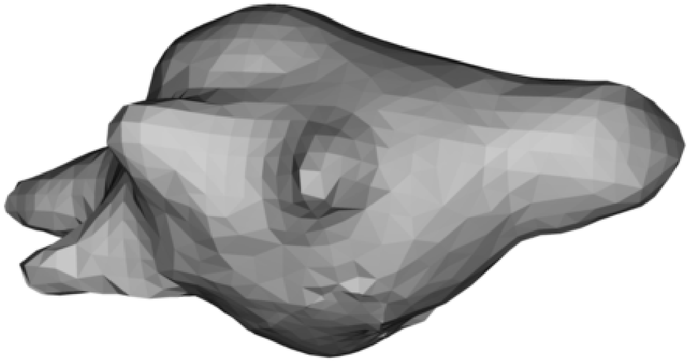

Oklahoma
N=1790
N=15
498
484
203
152
155
77
76
54
33
25
18
N.E. Colorado
N=2675
122
1137
626
223
112
108
53
106
81
56
38
13
1.00
0.90
0.80
0.70
0.60
0.50
AXIS RATIO
1-5
11-15
21-25
31-35
41-45
51-55
HAILSTONE MAXIMUM DIMENSION (mm)
I : 95% CONFIDENCE LEVEL

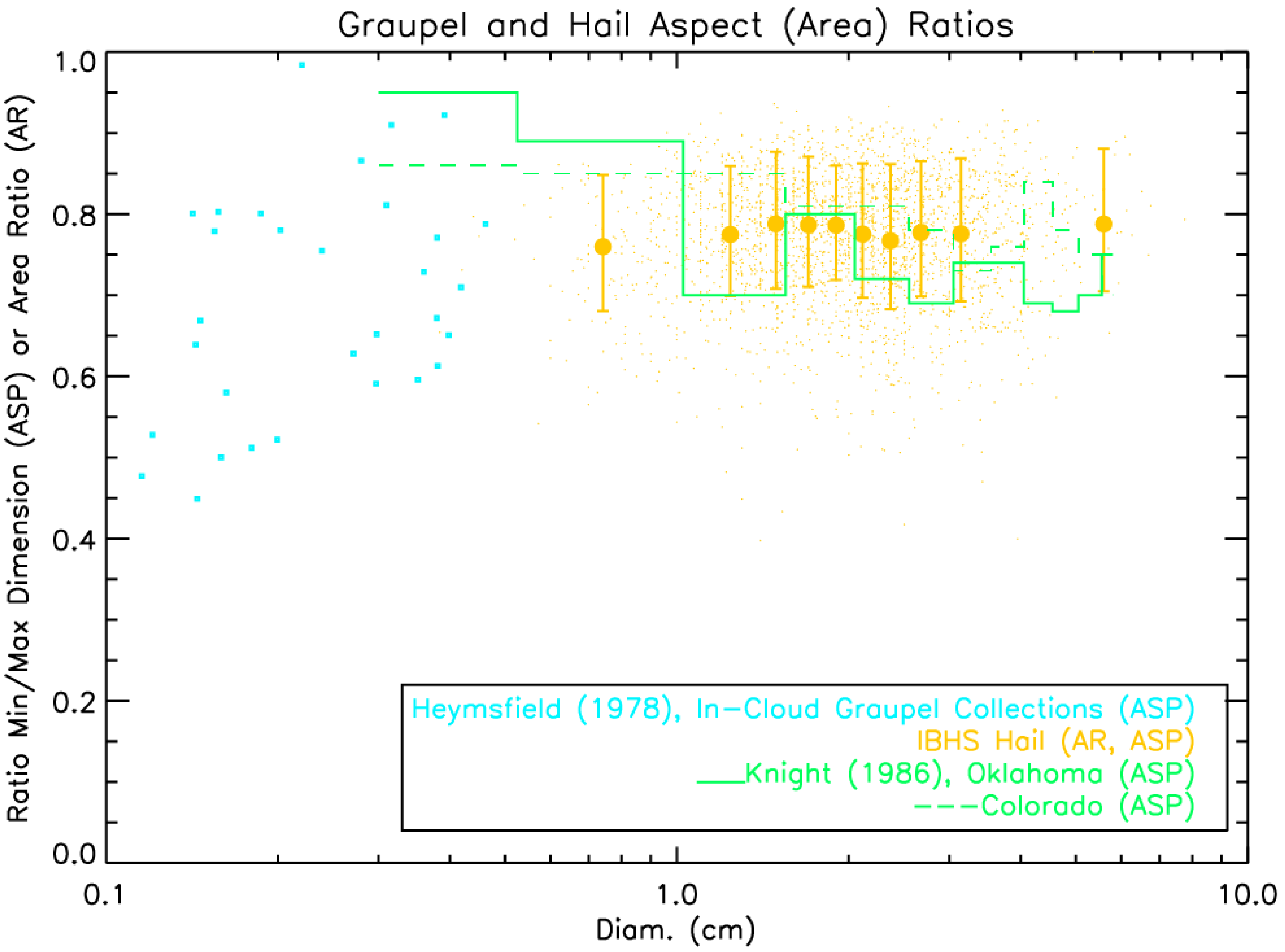
Knight (1986)

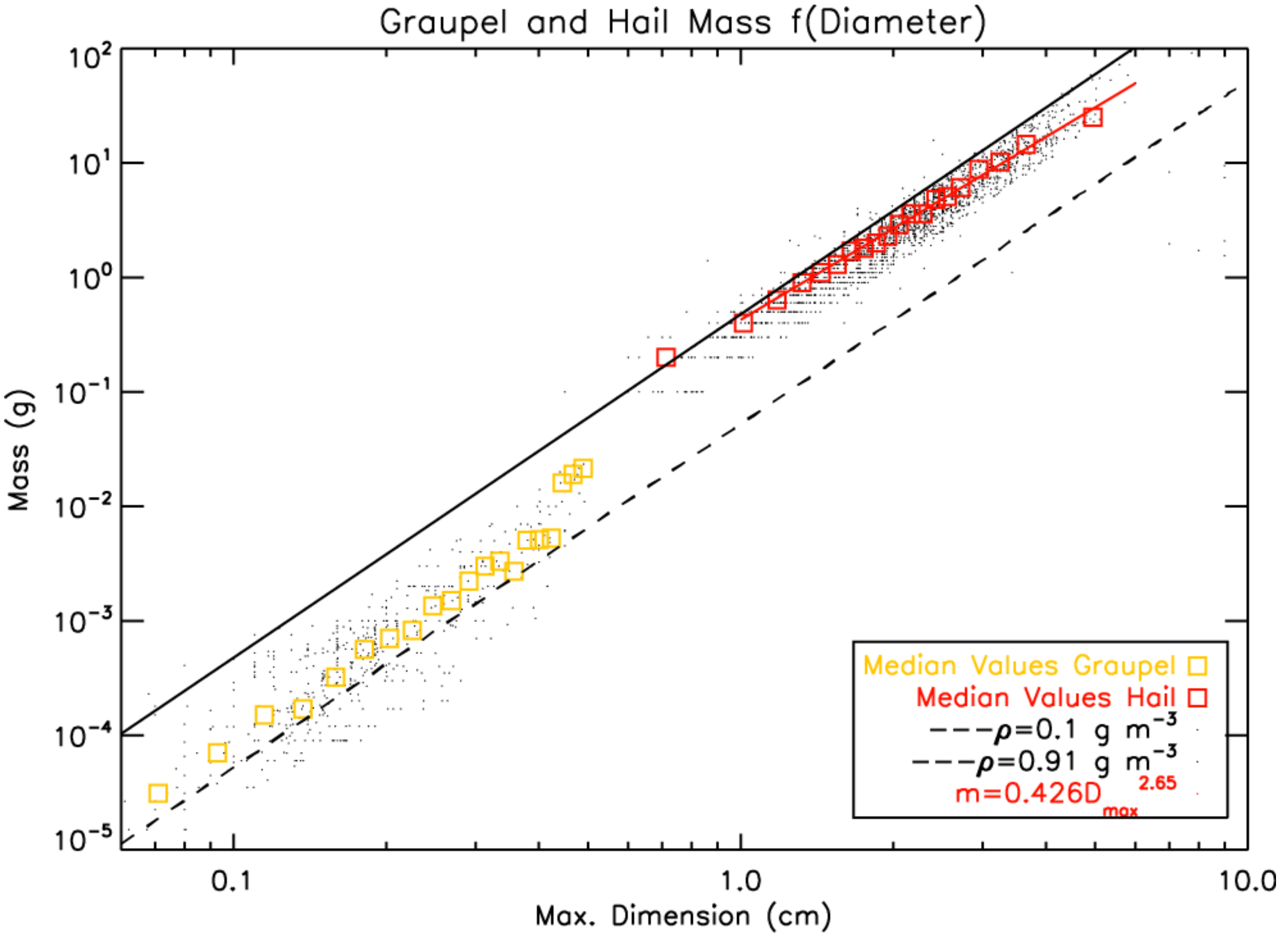


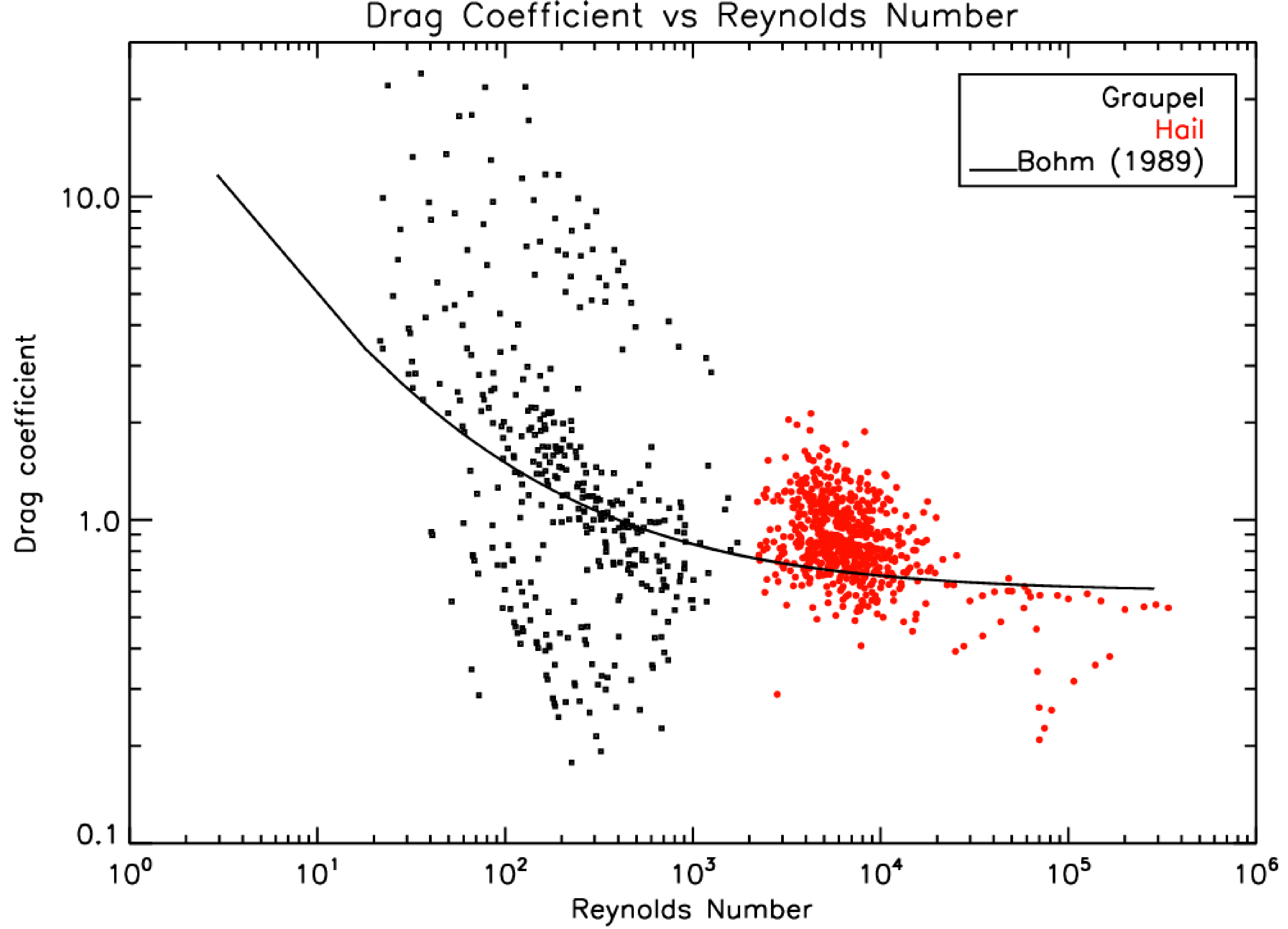
me of model = 0.254060 in³

Volume of model = 0.252218 in³

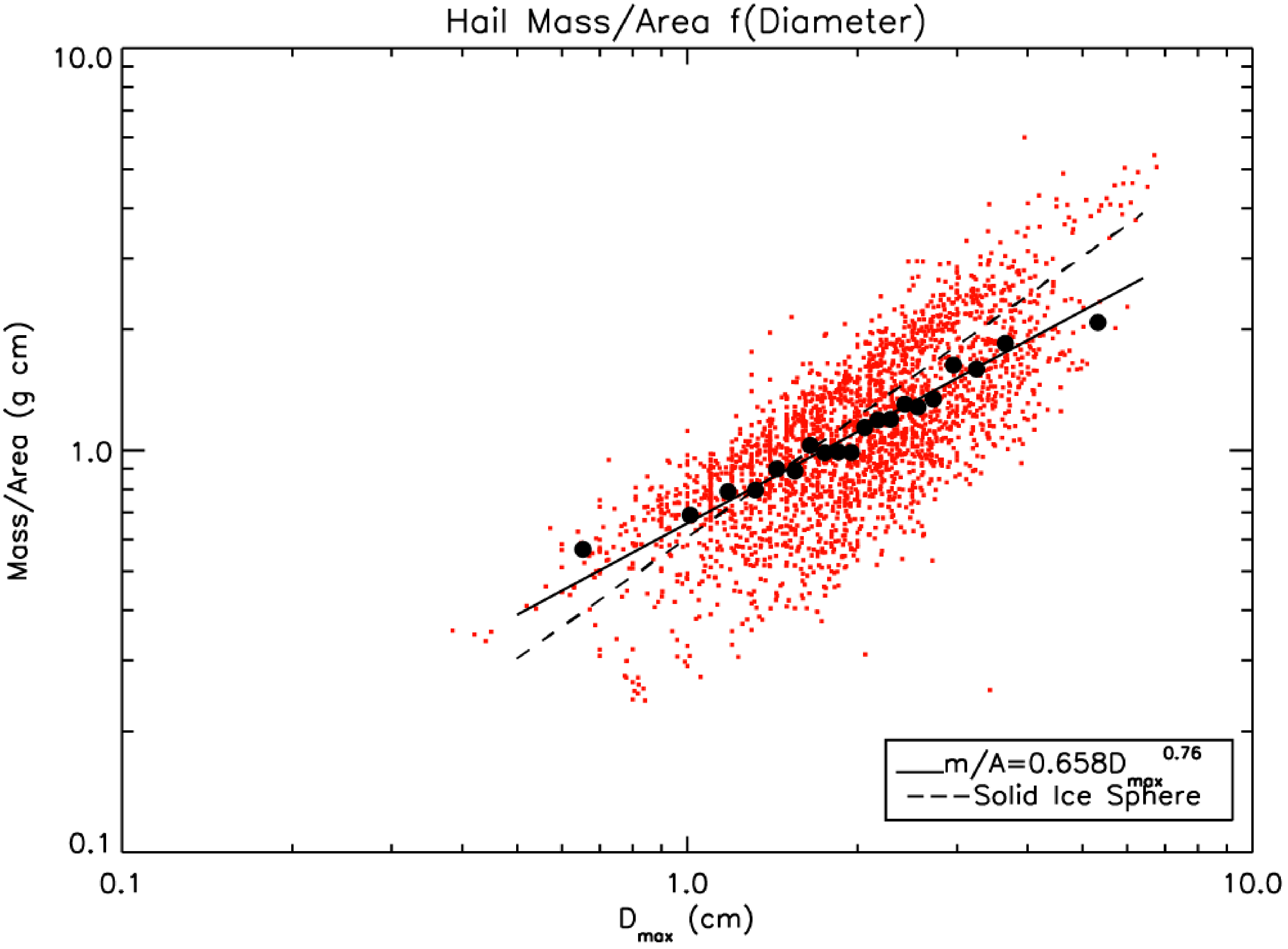


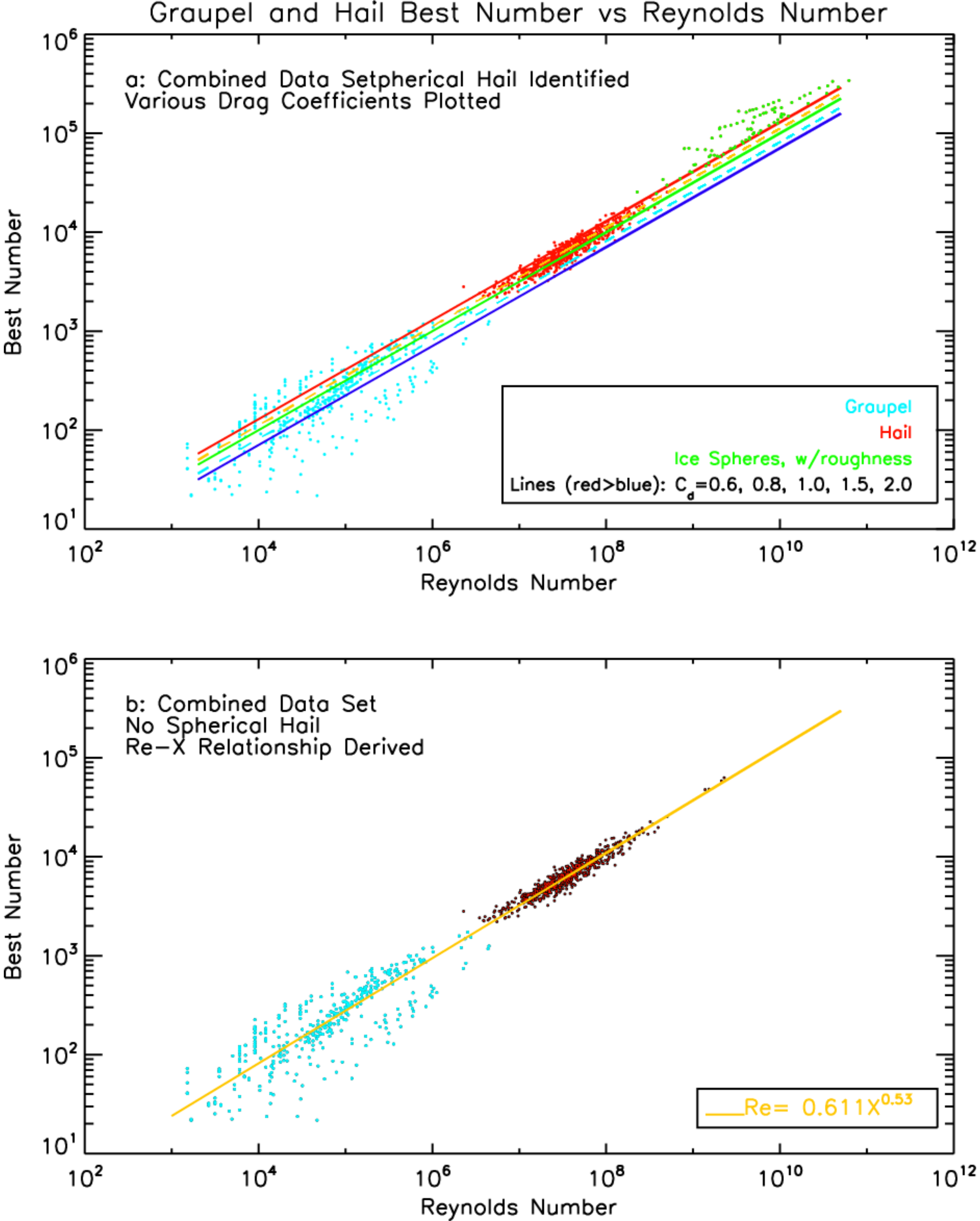

Graupel and Hail Aspect (Area) Ratios
Ratio Min/Max Dimension (ASP) or Area Ratio (AR)
Diam. (cm)
0.0
0.2
0.4
0.6
0.8
1.0
0.1
1.0
10.0
Heymsfield (1978), In-Cloud Graupel Collections (ASP)
IBHS Hail (AR, ASP)
Knight (1986), Oklahoma (ASP)
Colorado (ASP)


Graupel and Hail Mass f(Diameter)
Mass (g)
Max. Dimension (cm)
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
0.1
1.0
10.0
Median Values Graupel □
Median Values Hail □
---ρ=0.1 g m^-3
---ρ=0.91 g m^-3
m=0.426D_max^2.65


Drag Coefficient vs Reynolds Number
Graupel
Hail
Bohm (1989)
Drag coefficient
10.0
1.0
0.1
Reynolds Number
10^0
10^1
10^2
10^3
10^4
10^5
10^6

$$V_t = (2mg / \rho_f C_d A)^{0.5}$$




Graupel and Hail Best Number vs Reynolds Number
a: Combined Data Setpherical Hail Identified
Various Drag Coefficients Plotted
Graupel
Hail
Ice Spheres, w/roughness
Lines (red>blue): C_d=0.6, 0.8, 1.0, 1.5, 2.0
Best Number
Reynolds Number
10^1
10^2
10^3
10^4
10^5
10^6
10^2
10^4
10^6
10^8
10^10
10^12
b: Combined Data Set
No Spherical Hail
Re–X Relationship Derived
Re= 0.611X^0.53
Best Number
Reynolds Number
10^1
10^2
10^3
10^4
10^5
10^6
10^2
10^4
10^6
10^8
10^10
10^12


INCH
1
2
3
4
5
6
CHINA
STAINLESS STEEL
B
22
21
20
19
18
17
16

▶

Summary and Conclusions

- The relationship between the drag coefficient or the Best number and Reynolds number for graupel and hail are fairly well characterized
  - Laboratory studies will be used to evaluate this statement
- The critical issue is to identify graupel and hail masses, cross-sectional areas, and their ratio, for a range of storm types, formation mechanisms, and geographical locations.
- The new results can be readily used in weather forecast models