Return Times, Dispersion and Memory in Extremes of Mid-latitude Vorticity

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SYNOPSIS

Motivation	Non-Poisson behaviour of mid-latitude cyclones
Idea	Fractional Poisson processes
Data	ERA interim vorticity extremes (850 hPA)
Results	Weibull distribution of return times
	Dispersion
	Memory

(Blender et al. QJRMS, 2015)

Mid-latitude Cyclones

Mailier et al. (2006)
Cyclone tracks in ONDJFM
vorticity 850hPa (NCEP/NCAR)

Monthly means in 53 winters
5°grid
Dispersion

$$\Psi = \frac{\sigma_n^2}{\bar{n}} - 1$$

Genesis/Land: ψ ≈ 0
Exits/Lysis: ψ ≈ 0.5
Non-Poisson

Mechanisms:
families, parent cyclones
Serial correlation

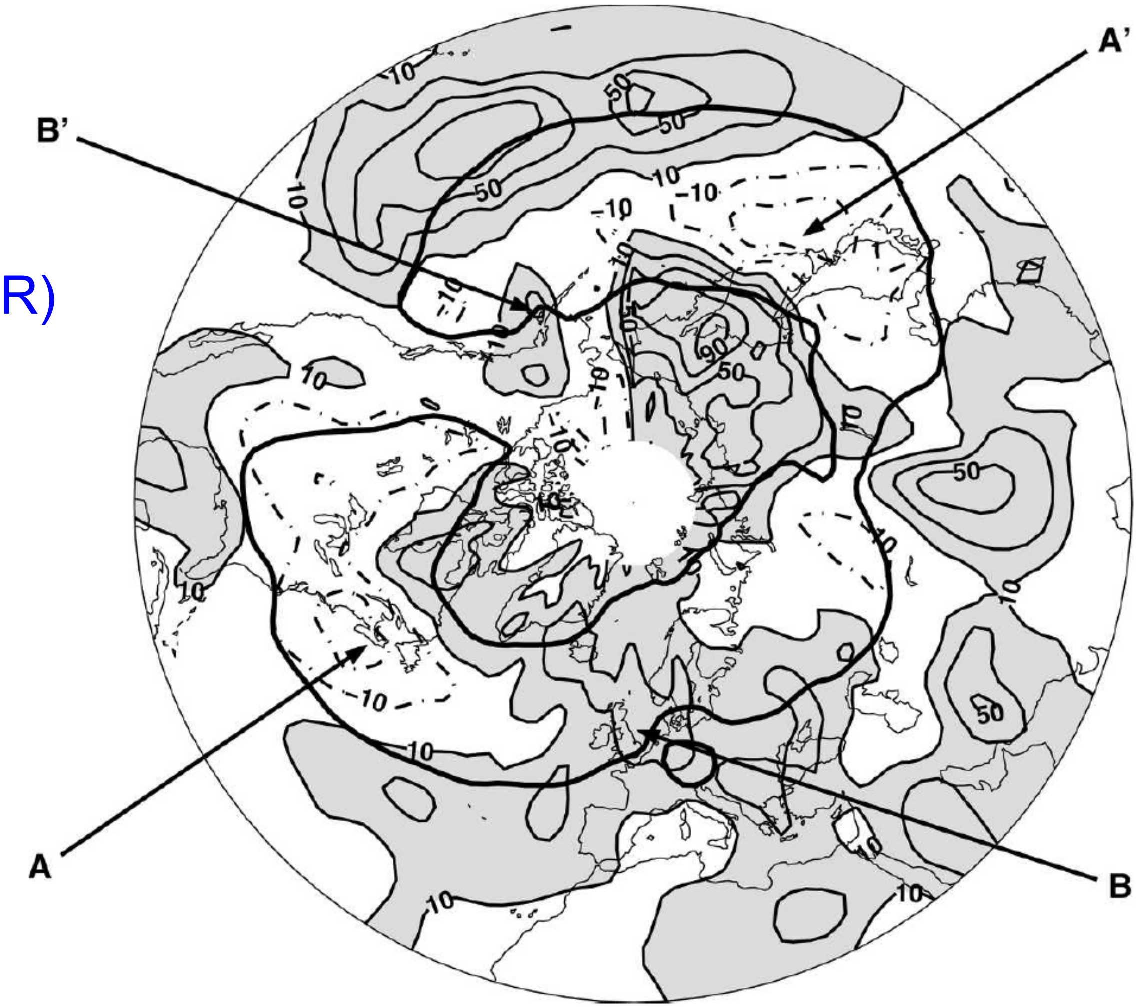


FIG. 5. Estimated dispersion statistic $\hat{\psi}$ (%) of the monthly number of cyclone transits. Solid (dashed) lines indicate positive (negative) values. Contours start at ±10%, contour intervals of 20%. Areas where $\hat{\psi} \geq 10\%$ are shaded. Thick dark lines representing the boundaries of the regions where $\overline{n} \geq 5$ month^{-1} (shaded areas in Fig. 3) have been added for easy reference to the storm tracks. For annotations A, B, A′, B′ see text.

Idea: Fractional Poisson Processes

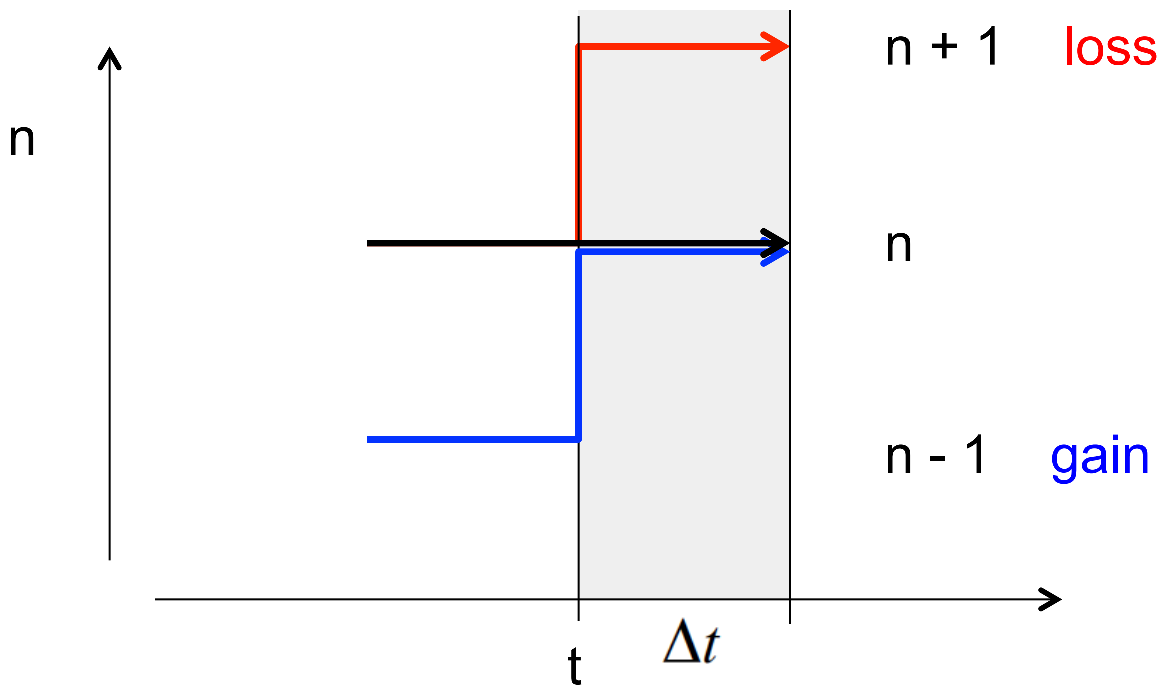
With memory

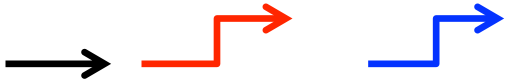
Laskin (2003): Mathematical Basis

Haubold et al. (2011): Review, Applications

The Poisson process time step for *P*(*n*,*t*)

Rate $\overline{n}$, Prob for event in Δt is $\overline{n}\Delta t$





$$P(n, t + \Delta t) = P(n, t)(1 - \overline{n}\Delta t) + P(n - 1, t)\overline{n}\Delta t, \quad n \geqslant 1$$

Time evolution

$$\frac{\partial P(n,t)}{\partial t} = \overline{n}(P(n-1,t) - P(n,t)), \quad n \geqslant 1$$

Fractional Poisson Process

$${}_0D_t^{\mu} P_{\mu}(n,t) = \nu\big(P_{\mu}(n-1,t) - P_{\mu}(n,t)\big) + \frac{t^{-\mu}}{\Gamma(1-\mu)}\delta_{n,0}, \quad 0 < \mu \leqslant 1$$

Fractional Kolmogorov–Feller equation (Laskin, 2003)

'fractional exponent' μ

Riemann–Liouville fractional integral

$${}_0D_t^{\mu} f(t) = \frac{1}{\Gamma(-\mu)} \int_0^t \frac{\mathrm{d}\tau\, f(\tau)}{(t-\tau)^{1+\mu}}$$

Mean in FPP

$$\overline{n}_\mu = \sum_{n=0}^{\infty} n P_\mu(n,t) = \frac{\nu t^\mu}{\Gamma(\mu+1)}$$

Variance

$$\sigma_\mu^2 = \overline{n}_\mu + \overline{n}_\mu^2 \left(\frac{\mu\Gamma(\mu)\Gamma(1/2)}{2^{2\mu-1}\Gamma(\mu+1/2)} - 1 \right)$$

Poisson $\mu = 1$

Intensity, rate $\nu \longrightarrow \overline{n}$

Mean $\overline{n}_1 = \overline{n} t$

Waiting time probability distribution function for FPP
Mittag-Leffler function (1903)

$$\psi_\mu(\tau) = v\tau^{\mu-1}E_{\mu,\mu}(-v\tau^\mu), \quad t \geqslant 0, \quad 0 < \mu \leqslant 1$$

Limiting cases

$$E_{1,1}(x) = e^x \qquad\qquad E_{1/2,1/2}(x) = e^{x^2}(1 + \mathrm{erf}(x))$$

Idea: Approximate ML by exp, yields Weibull pdf

$$p(t, \lambda, k) = \frac{k}{\lambda}\left(\frac{t}{\lambda}\right)^{k-1} e^{-(t/\lambda)^k}$$

Shape parameter $k = \mu$

Waiting time distribution

$$\psi_{\mu}(\tau) = \nu\tau^{\mu-1}E_{\mu,\mu}(-\nu\tau^{\mu}), \quad t \geqslant 0, \quad 0 < \mu \leqslant 1$$

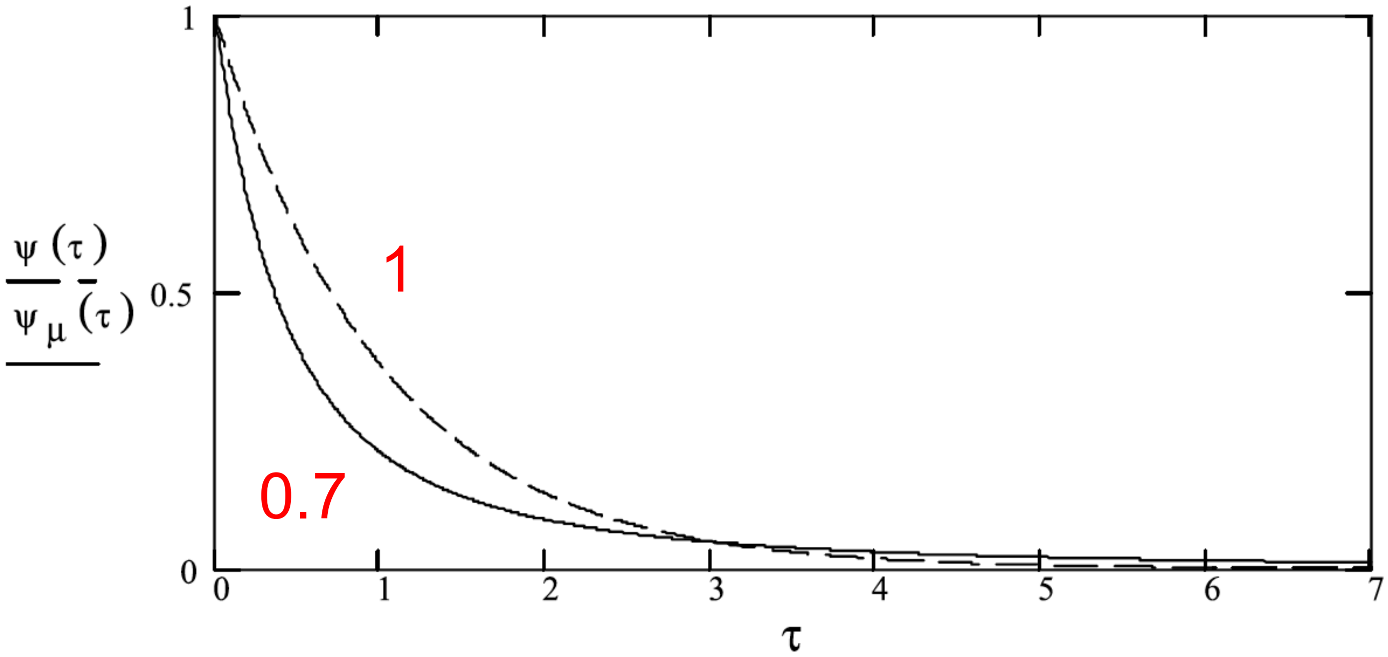


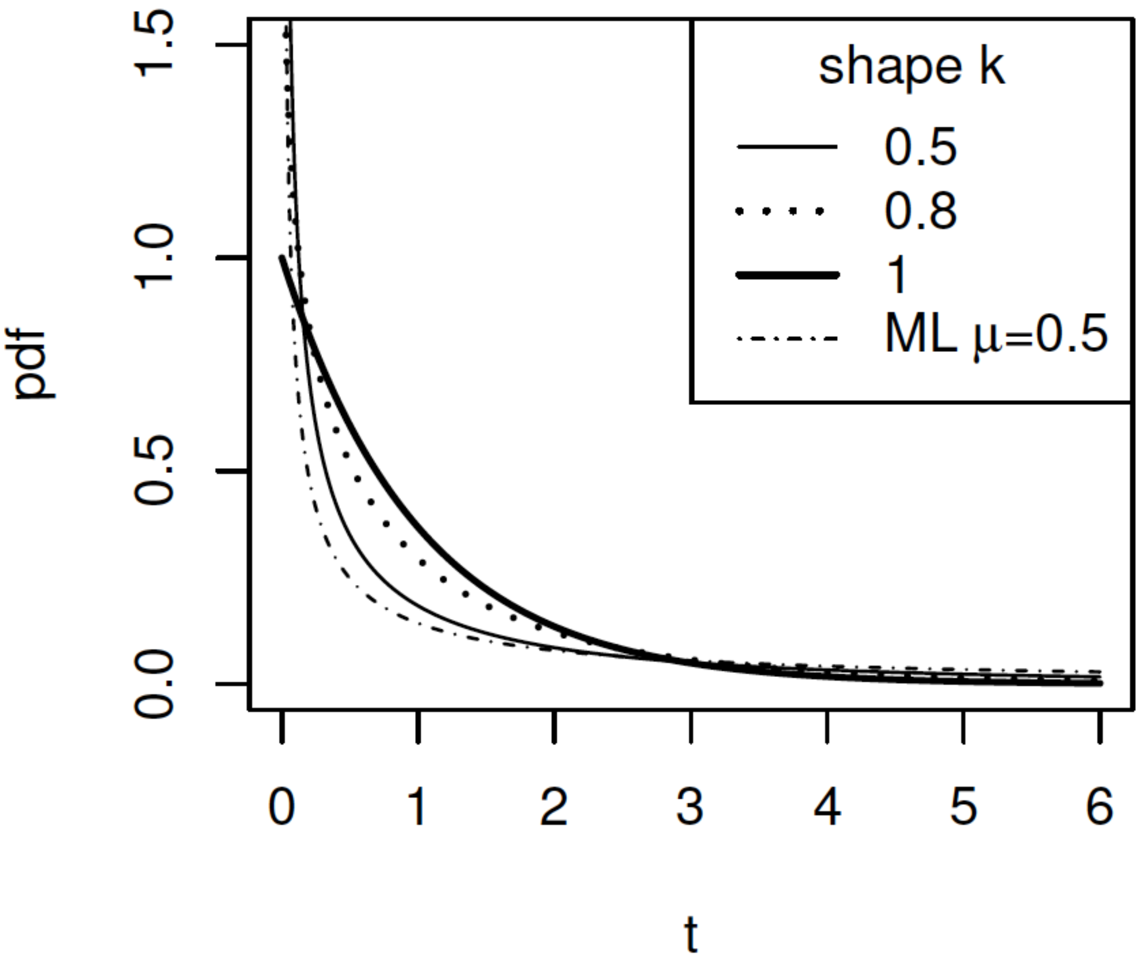
Fig. 1 (Laskin, 2003): Waiting time distribution

$$\overline{n} = 1,\ \nu = 1,\ \mu = 0.7$$

Approximation of waiting time distribution by

Weibull with shape parameter k, data: k = 0.5 ...1

Advantages: Standard routines available, relation to memory



Review on applications of fractionally differenced models and the Mittag-Leffler function: Haubold (2011)

Time-fractional diffusion equation

$$\frac{\partial^\alpha}{\partial t^\alpha} N(x,t) = D \frac{\partial^2}{\partial x^2} N(x,t), \quad 0 < \alpha < 1$$

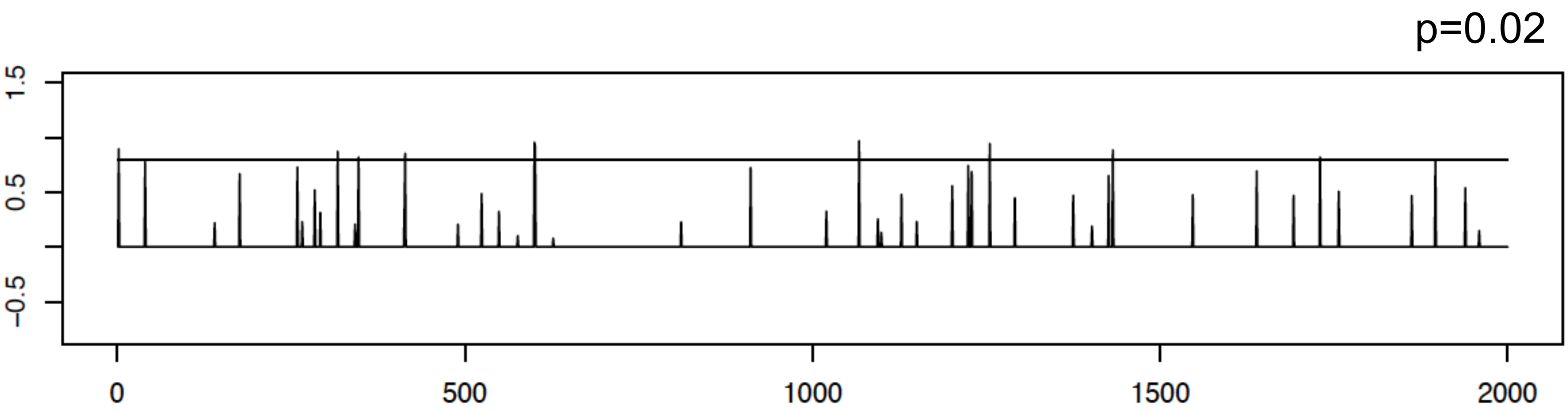
Fractional space diffusion eqution

$$\frac{\partial}{\partial t} N(x,t) = D \frac{\partial^\alpha}{\partial x^\alpha} N(x,t), \quad 0 < \alpha < 1$$

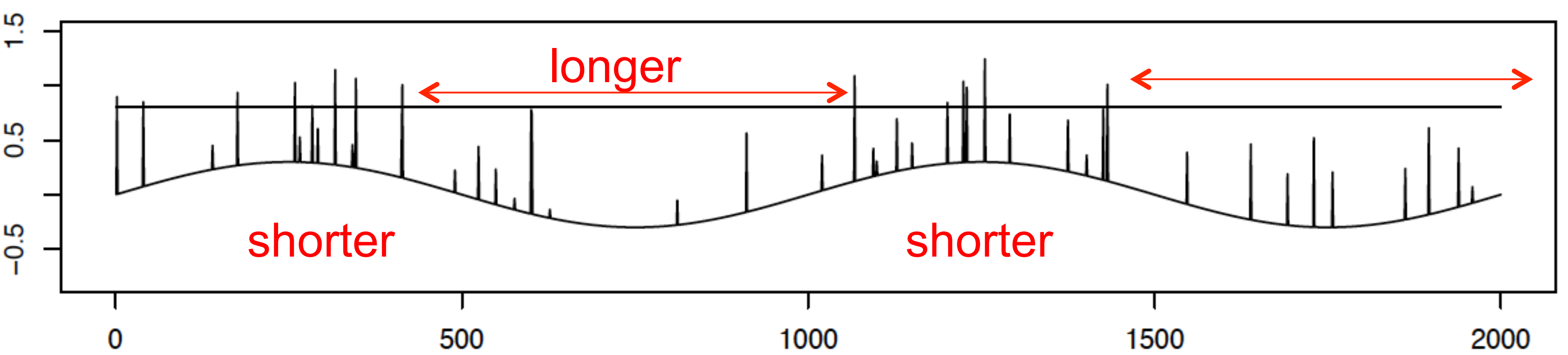
The Mechanism

Memory and non-exponential return times

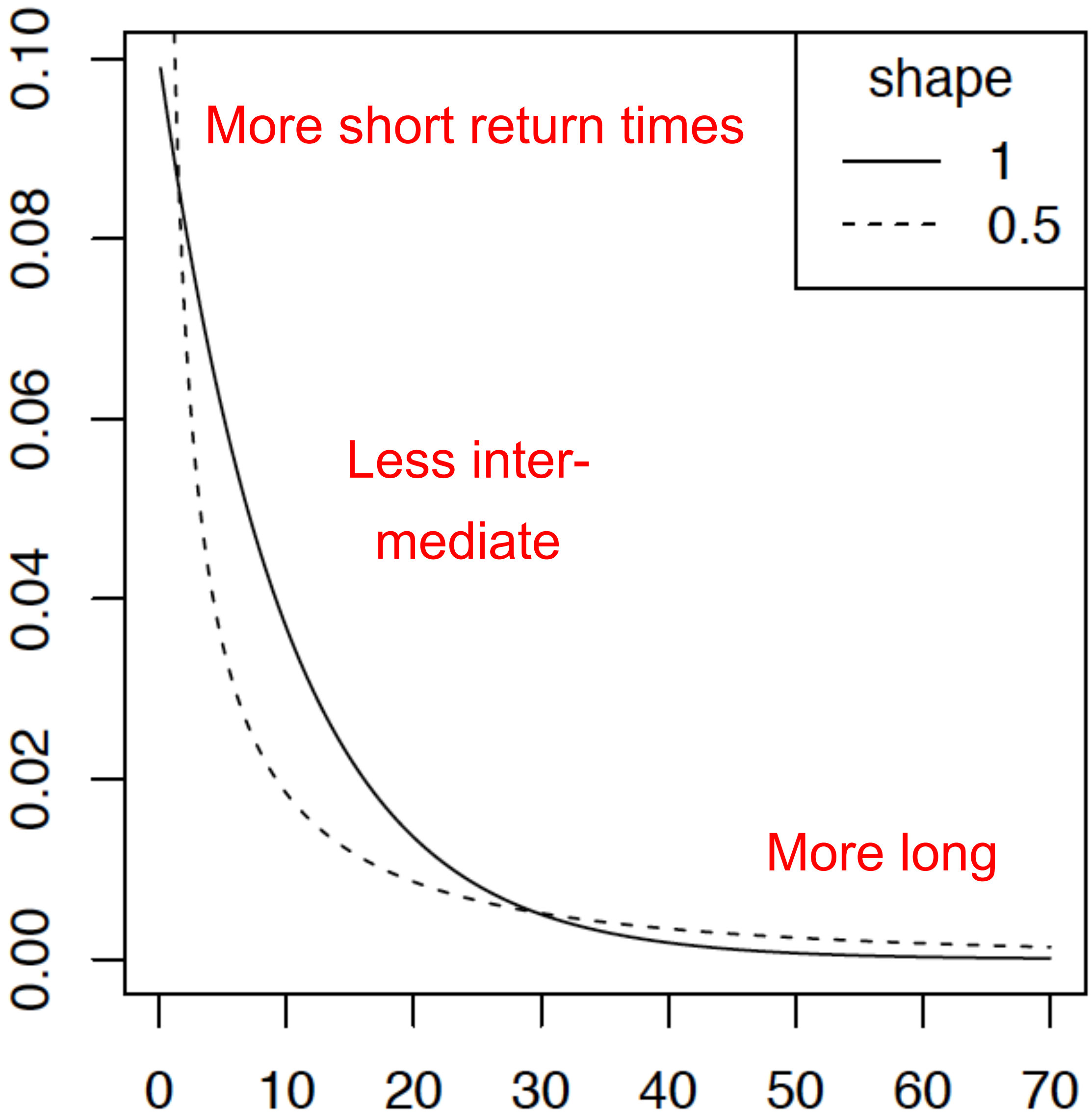
Rare events, with random height



Changes of return times with memory

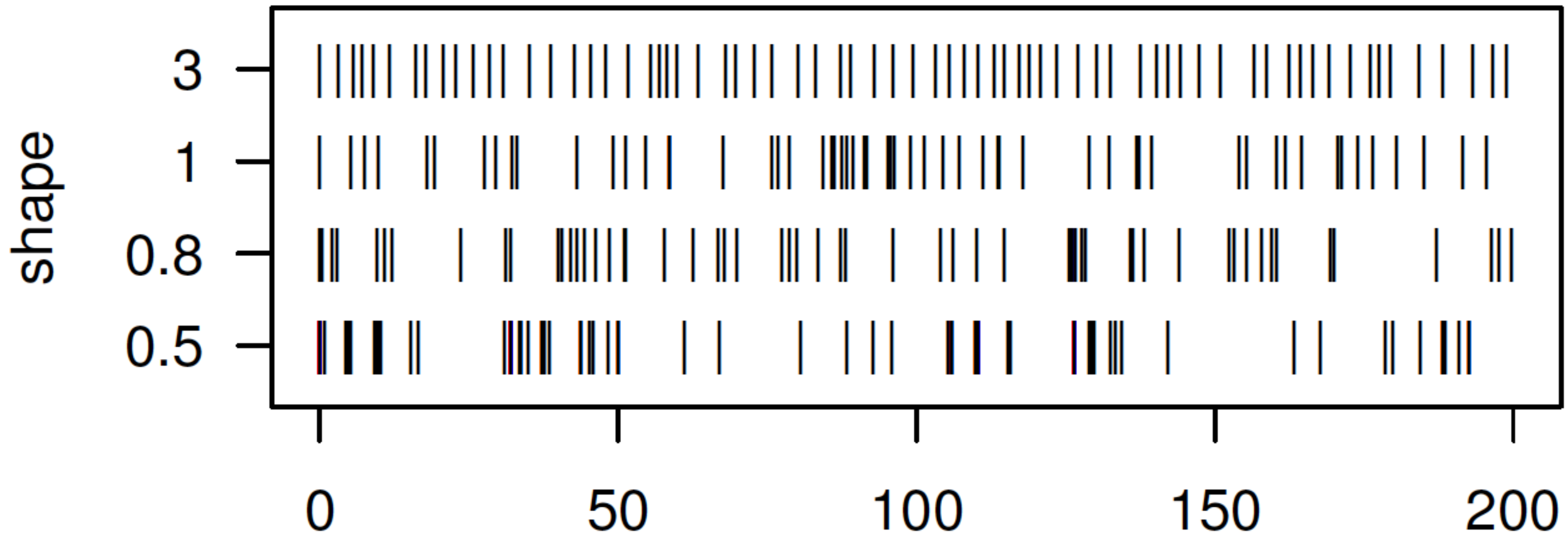


Weibull distribution



Scale = 10

Time series of events with Weibull distributed return times
No correlation



k = 3 quasi-regular, for comparison

k = 1 exponential, standard Poisson process

k = 0.5, 0.8 found in data, increased clustering

Compare Poisson with FPP (Cahoy et al., 2010)

Fig. 1. Sample trajectories of: (a) standard Poisson process, (b) fPp with parameter $\nu = 1/2$.

Estimation of FPP parameters: Cahoy et al. (2010)

Results for ERA interim

Vorticity Extremes

Relative vorticity 850 hPa

1.5° x 1.5°, 6h

Winters DJF, Summers JJA

1980-2013

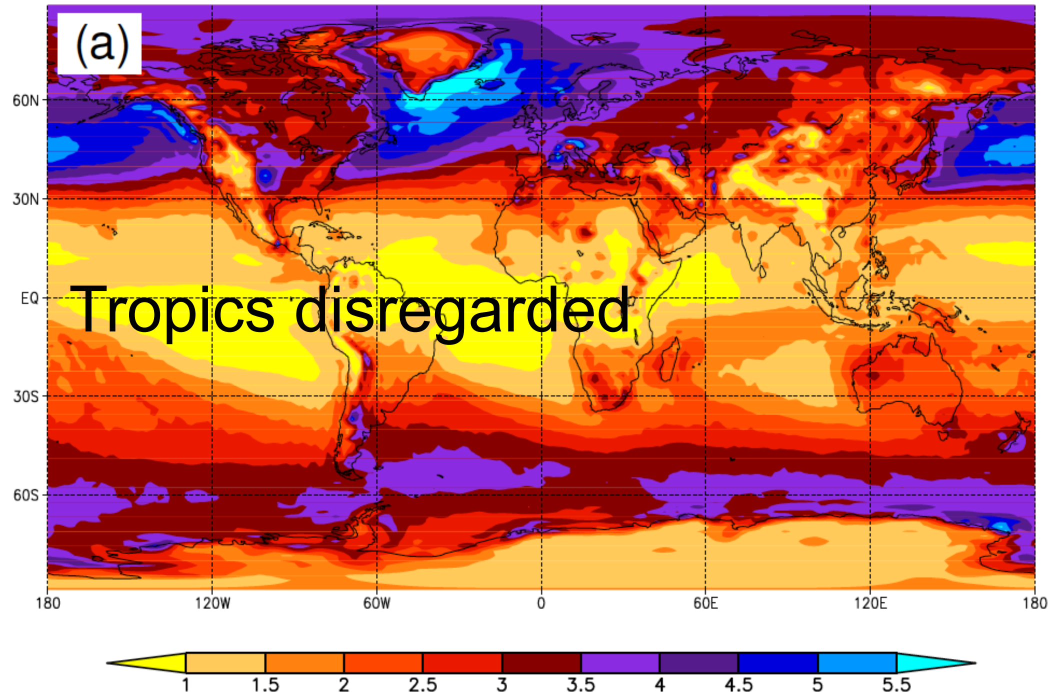
Standard Deviation

ERA interim

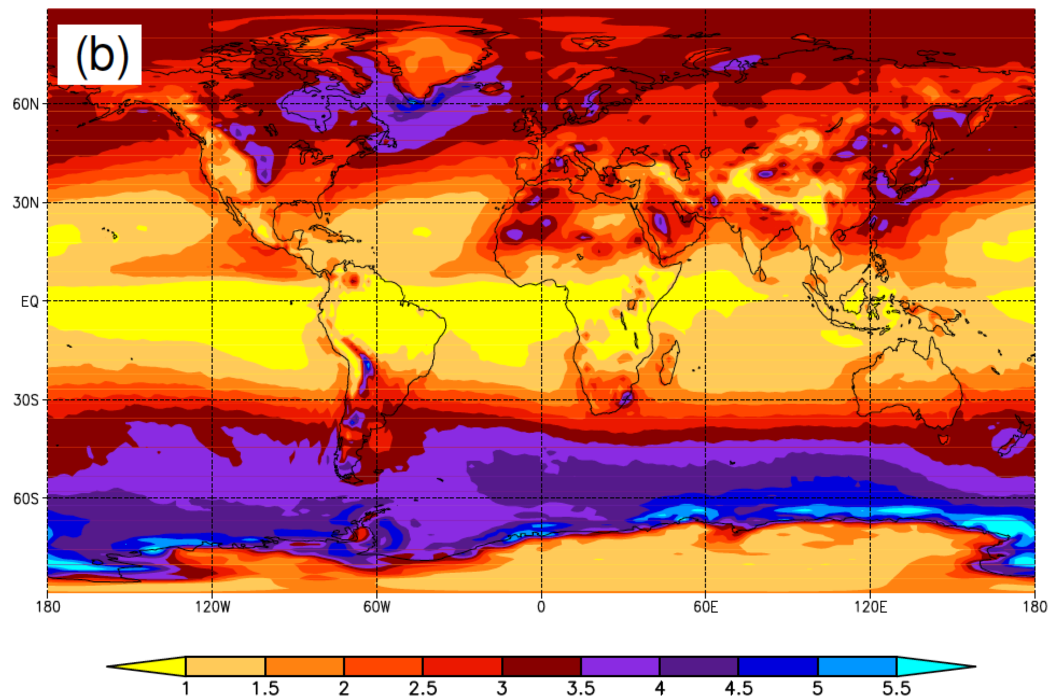
Relative vorticity 850 hPa

Indicates mid-latitude storm tracks

DJF



JJA

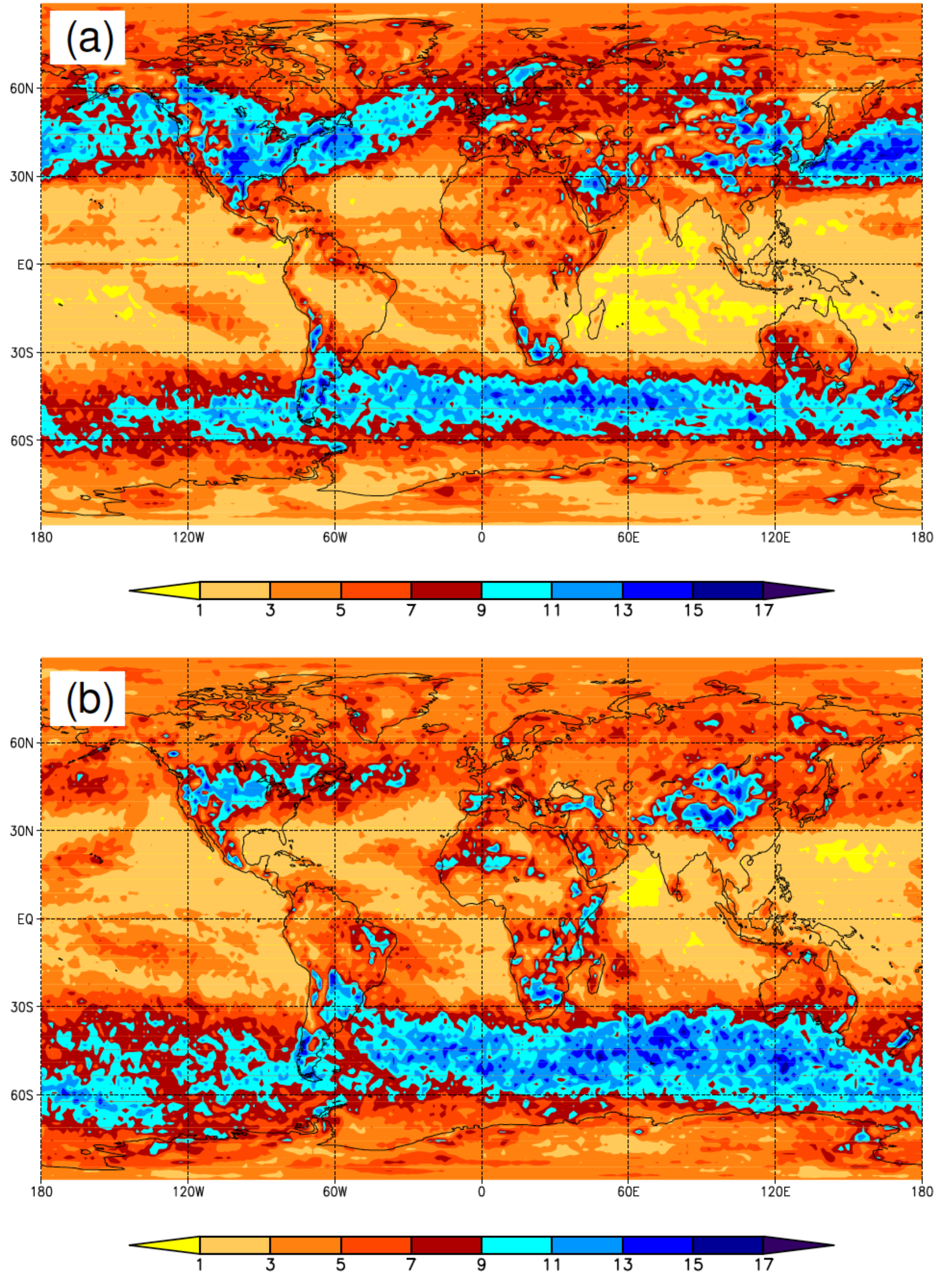


Scale parameter λ
[days]

Extremes 99% quantile
Weibull fit to return times
(exclude 6h)

$$p(t, \lambda, k) = \frac{k}{\lambda}\left(\frac{t}{\lambda}\right)^{k-1} e^{-(t/\lambda)^k}$$

Long scales in genesis regions of storm tracks



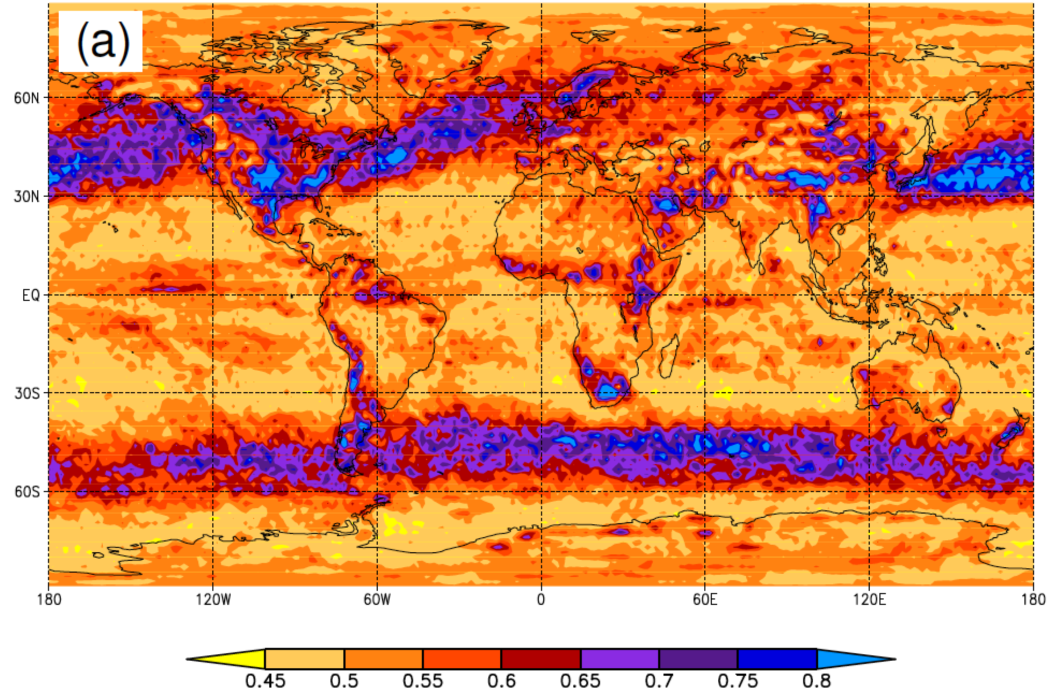
Shape parameter k

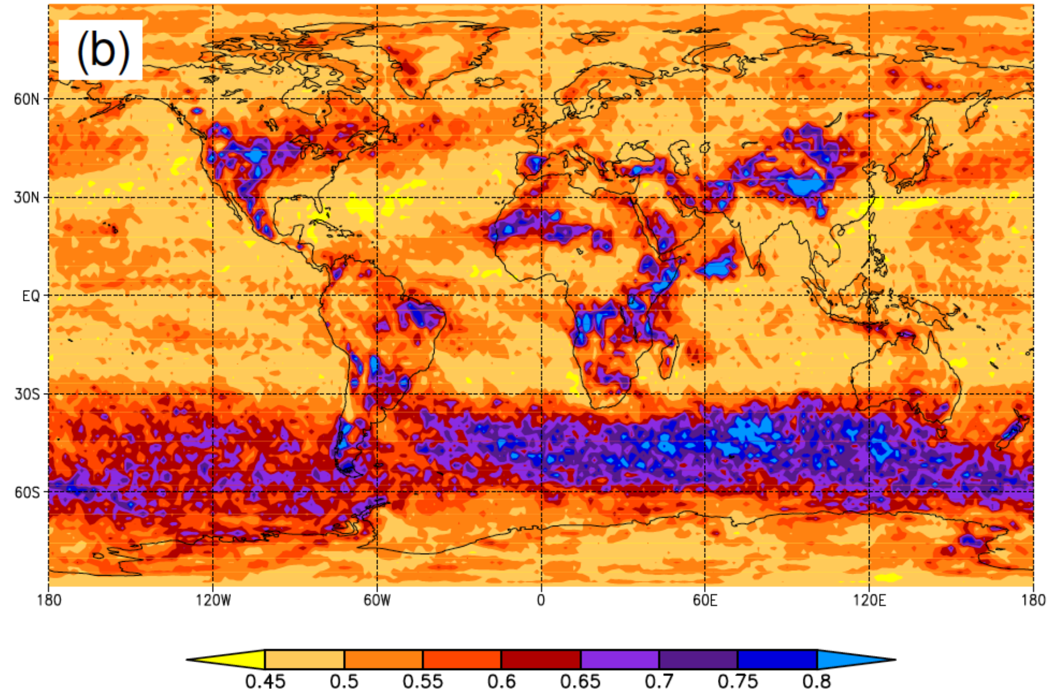
Weibull

$$k = \mu$$

Shape parameters

Lysis/exits ≈ 0.5
Genesis ≈ 1





Dispersion for FPPs

$$\psi = \sigma_\mu^2/\bar{n}_\mu - 1$$

Mean and var FPP

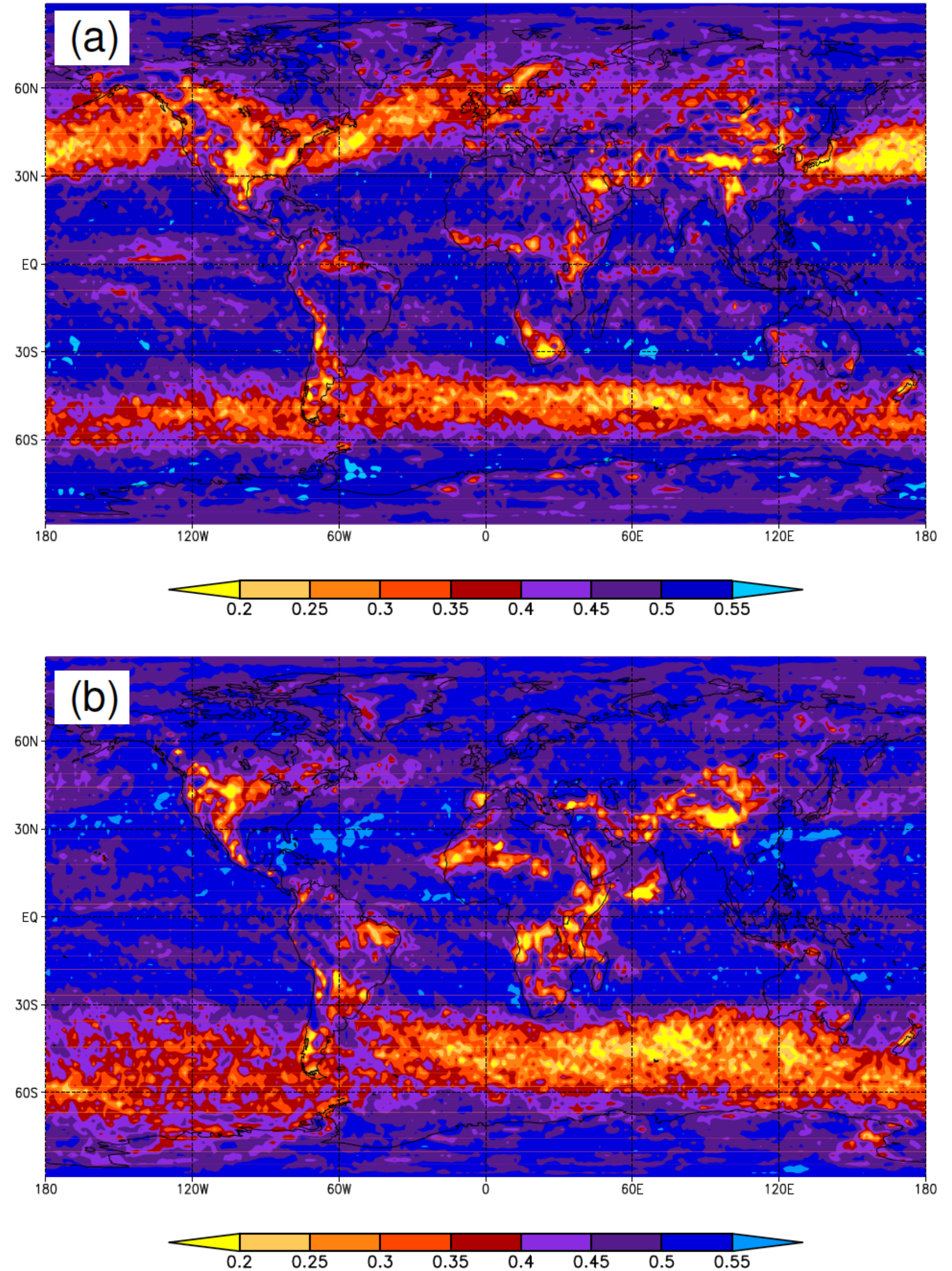
$$\overline{n}_\mu = \frac{\nu t^\mu}{\Gamma(\mu+1)}$$

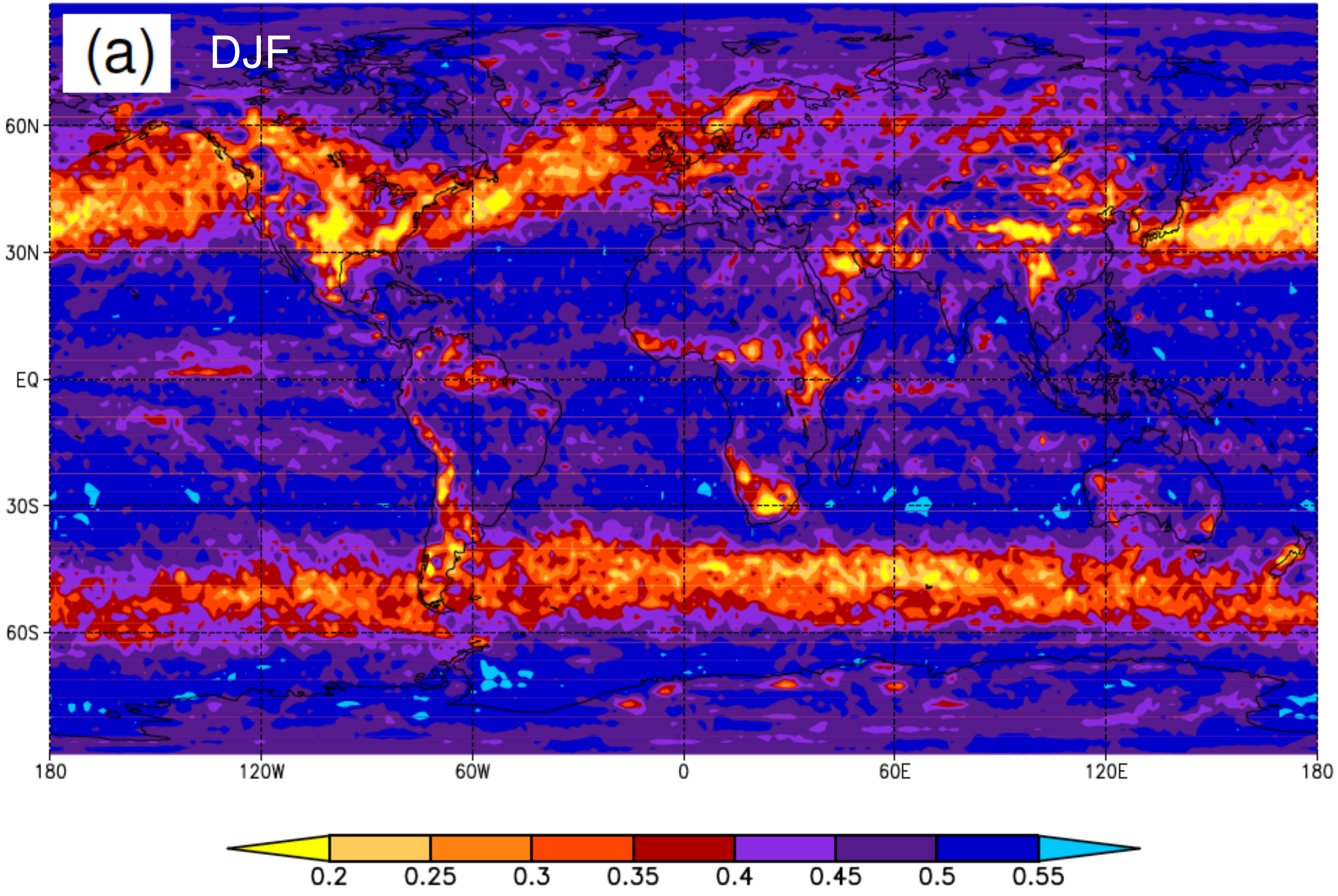
$$\sigma_\mu^2 = \overline{n}_\mu + \overline{n}_\mu^2 \left(\frac{\mu\Gamma(\mu)\Gamma(1/2)}{2^{2\mu-1}\Gamma(\mu+1/2)} - 1 \right)$$

Dispersion from shape k≈1

$$\psi/\nu t \approx 1 - k$$

Cyclogenesis: Poisson
Exit/lysis: FPP
Mailier et al. also: ψ ≈ 0.5





ψ increases along the storm track axis

Long-term memory and extremes

Finding/Hypothesis: Long-term memory leads to Weibull distributed return times

Correlation function $C(t) \sim t^{-\gamma}$ $\quad k \approx \gamma$

Bunde et al. (2003), Santhanam and Kantz (2008), Blender et al. (2008)

Determine LTM with Detrended Fluctuation Analysis (DFA)

Fluctuation function. Power-law fit in t = 5 – 30 days

$$F(t) \sim t^{\alpha}$$

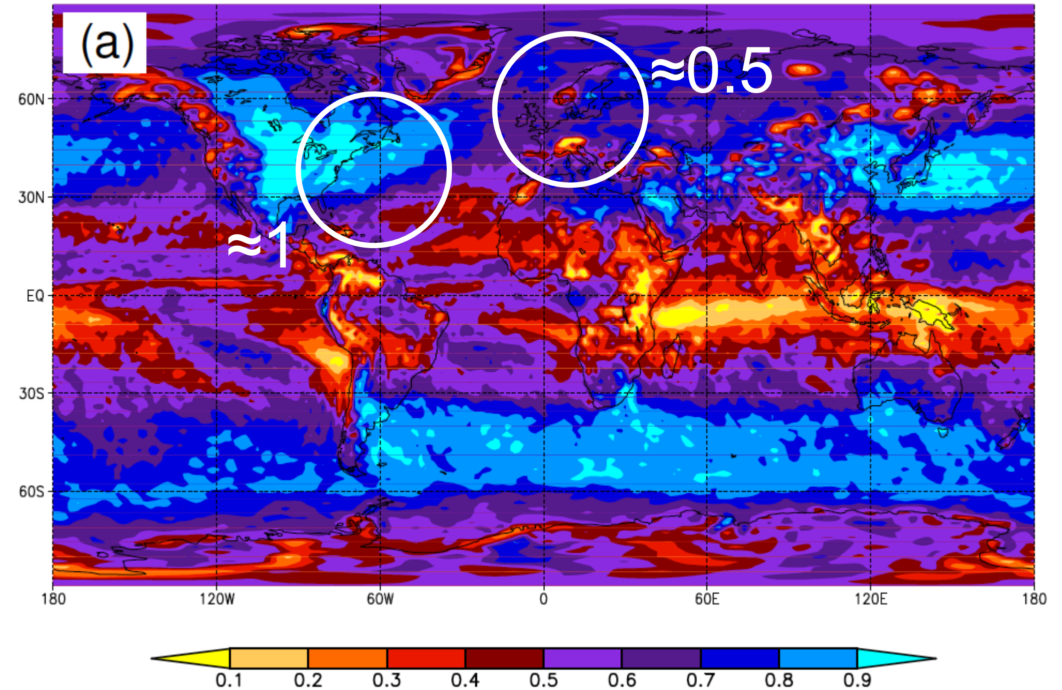
Power spectrum

$$S(f) \sim f^{-\beta} \qquad \beta = 2\alpha - 1 = 1 - \gamma$$

Shape parameter

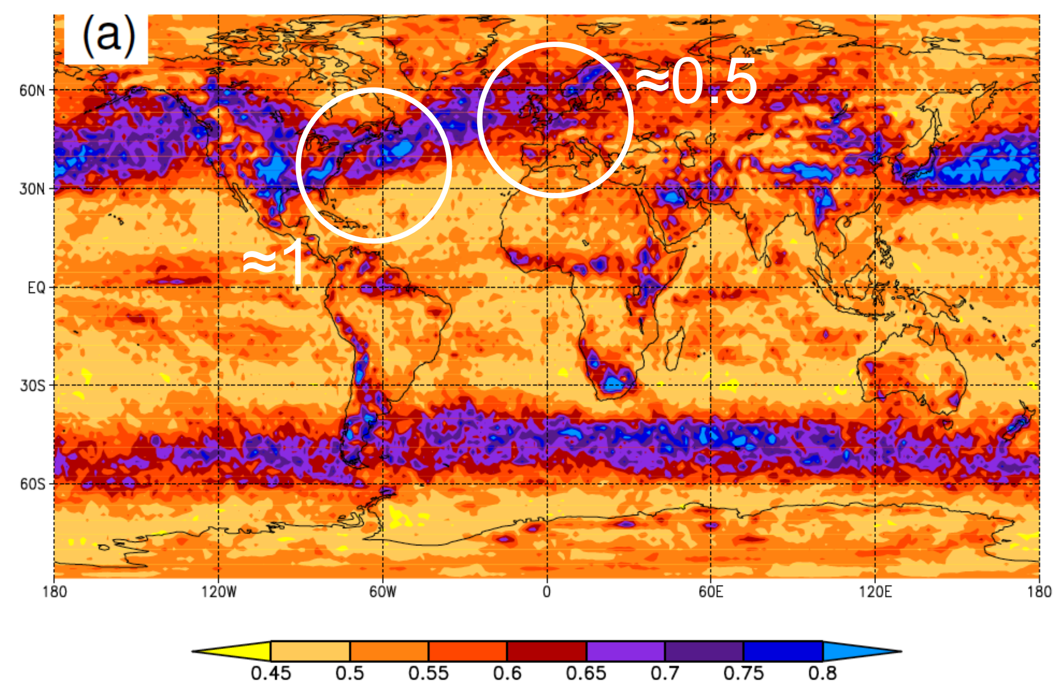
Use DFA result to predict

$$k_{LTM} = 1 - \beta$$



Compare Weibull fit

Decay along storm track



SUMMARY

Return time distribution of extremes in 850hPa vorticity

FPP return times: approximated by Weibull

Cyclogenesis uncorrelated (Poisson Process)

Storm track exit/lysis correlated (Fractional P. P.)

A single FPP parameter μ explains

- Non-exponential return times (Weibull, shape $k \approx \mu$)
- Dispersion $\psi \sim 1 - k$
- Correlation, spectra: $\beta \approx 1 - k$

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